Using Portfolios in an Introduction to Proofs Course

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Motivation

- Wanted to give students lots of practice with proof writing
- Wanted to incorporate the process of proof revision
- Didn’t want grading to be too onerous

Course Population

- Typically 10-25 students per semester
- Sophomore gateway course for math and secondary ed math majors
- Also taken by chemistry and engineering majors to complete math minor
- Main goal of course is for students to learn to write basic mathematical proofs

Course Topics

- Proof techniques (existence theorems, direct proof, contrapositive, contradiction, induction)
- Number theory (divisibility, odds, evens, primes, composites, rationals, irrationals)
- Set theory (subsets, complements, unions, intersections)
- Functions (one-to-one, onto, composition, inverses)

Course Structure

- 20 % - Weekly proof first drafts
- 10 % - Participation
- 40 % - 4 in-class exams
- 30 % - Proof portfolio

Weekly Proof First Drafts

- 5-7 proofs assigned per week
- Students work on outside of class
- Students also work in groups and get help from instructor during “proof workshop” days
- 1-2 proofs collected and graded
- Each student receives detailed comments on how to improve his or her specific proof

Proof Grading Scale

- All proofs throughout the course are graded out of 10 points on the following scale:
  - 10 – solution completely correct
  - 8 – solution mostly correct with some minor mistakes
  - 6 – main idea of solution is correct with some moderate gaps
  - 4 – main idea of solution is missed, but some good ideas are present
  - 2 – minor progress toward a correct solution
  - 0 – nothing done toward a correct solution

Participation

- Largely based on presentations of weekly assigned proofs on day they are due
  - Usually ask for 2-3 volunteers to present some of the assigned proofs for the week
  - Class discusses and critiques presented proofs
  - Not graded for correctness
- Also based on student attendance and discussion during class

4 In-Class Exams

- First exam on logic
- Other 3 exams on proofs
  - 5 problems
  - 3 are proofs students have already seen and received feedback on (examples done during lecture, or weekly assigned proofs that were collected or presented)
  - 2 are new proofs (straightforward ones)
  - Again each student receives detailed comments on how to improve his or her specific proofs

Proof Portfolio

- At the end of semester instructor selects 10 proofs representing all the different types encountered throughout semester
- Proofs are selected from those for which students have received feedback
  - Weekly graded proofs
  - Presented proofs
  - Proofs on exams

Proof Portfolio cont'd.

- Each selected proof includes
  - First draft
  - Revised draft
  - Explanation of revisions
- Revised drafts are graded on same 10 point scale
- Explanations are graded
- Portfolios also graded for style

Advantages

- Gets students to practice writing lots of proofs (probably around 50 throughout the semester)
- Gives students feedback without having to grade every single proof
- Teaches students to build on their work, not to look for a model “correct” proof
- Students are surprised to see how much they have improved at end of semester

Disadvantages

- Students often wait for “proof workshop” days to work on proof
- Students present complete proofs that are usually mostly correct, so there is often very little discussion
- Students don’t reflect on and revise proof first drafts until end of semester

Future Plans

- Have students present proofs before they are due to spark more discussion
- Try to employ peer critique to give students feedback on even more proofs
- Try to integrate logic throughout course
- Have students write revised proof drafts earlier in semester

Questions or Suggestions?