Teach What You Do: Providing an Authentic Mathematical Experience in an Introduction to Proofs Class

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Points-Free Grading

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- Encourages taking risks

Sums of Consecutive Integers

- Calculate $29 + 30 + 31$.
- Calculate $21 + 22 + 23 + 24$.
- Add up the numbers from 16 to 20. That is, calculate $16 + 17 + 18 + 19 + 20$.
- Add up the numbers from 6 to 14.
- Calculate the sums of other sequences of consecutive integers.
- What pattern(s) do you observe?
- Can you extend any of the patterns?
- How would you describe or explain the patterns?
- Is there anything special about any of the numbers?
- What questions do the patterns raise for you? What else would you like to know?

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- The number 90 can be written as a sum of consecutive integers in exactly four different ways.
- The number 90 can be written as a sum of consecutive positive integers in exactly four different ways.

Patterns in the Multiplication Table

Looking at the products in a 12×12 multiplication table:

- What patterns do you notice in how often numbers appear?
- Do you have any ideas about how you might predict how often a given number would appear in the table?
- What other questions do the patterns raise for you? What else would you like to know?

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- The product of any pair of twin primes is always one less than a perfect square multiple of 36.
- For any positive integer, $n > 1$, if the prime factorization of n is $n = p_1^{e_1} \cdots p_k^{e_k}$, then n has $(e_1 + 1) \cdots (e_k + 1)$ distinct positive integer factors.

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- Low-stakes assessment helps students be comfortable taking risks
- Conjecturing activities help students see proofs as natural, rather than as rituals
- Creating good opportunities for conjecturing is hard but definitely worth it!

References I

[1] Mike Brilleslyper et al. "What's The Point? The Benefits of Grading Without Points". In: *Primus* 22.5 (July 2012), pp. 411–427. ISSN: 1051-1970. DOI: 10.1080/10511970.2011.571346.

[2] G Harel and L Sowder. "Students' proof schemes: Results from exploratory studies". In: *Research in collegiate mathematics . . .* (1998).