Basic Probability

- What is probability?
- What must we know to determine probability?
- How do we determine probability?

What is probability?

- A measure of the likelihood of an *occurrence* or a chance behavior
- A number expressing the likelihood that an *event* occurs
- The ratio of the actual number of *occurrences* to the possible number of *occurrences*

What is an experiment?

- An experiment is an activity with an observable result.

- Examples of experiments:
  - Rolling a fair die
  - Picking a card
  - Picking a marble out of a jar for which the number and colors of the marbles is known

What is a trial?

- A trial is a performance or a repetition of an experiment.
- A trial results in an outcome.

Example – Tossing a Coin

- For the experiment of tossing a coin,
  - Each toss of the coin is a trial
  - There are two possible outcomes
    - The coin landing head side up – this is called *heads*
    - The coin landing tail side up – this is called *tails*

Example – Drawing a Card

- For the experiment of drawing a card from a fair 52-card poker deck
  - This experiment can be done *with replacement* – the card is put back into the deck
  - This experiment can be done *without replacement* – the card is not put back into the deck

Example – Drawing a Card

- For the experiment of drawing a card from a fair 52-card poker deck *with replacement*
  - The number of cards available in the deck from which one can choose a card is always the same, 52.

Example – Drawing a Card

- For the experiment of drawing a card from a fair 52-card poker deck *without replacement*
  - The number of cards available in the deck from which one can choose a card is decreases by one every time a card is taken from the deck

Example – Drawing a Card

- NOTE: For the experiment of drawing a card from a fair 52-card poker deck,
  - *With* or *without replacement* is not a concern if we draw only ONE (1) card from the deck

Example – Drawing a Card

- NOTE: For the experiment of drawing a card from a fair 52-card poker deck,
  - If we draw more than one card from the deck then we must know if the card is drawn *with* or *without replacement*

Example – Drawing a Card

- For the experiment of drawing a card from a fair 52-card poker deck,
  - A trial is drawing a card from the deck
    - Getting any of the 52 cards in the deck is a possible outcome, for example, getting the King of Diamonds (K♦)

Example – Rolling a Fair Die

- For the experiment of rolling a fair die,
  - A trial is rolling the fair die once
    - There are six (6) possible outcomes, having the die land displaying a 1, 2, 3, 4, 5, or 6 on the top face of the die.

What is an event?

- A *simple event* is a the occurrence of a single outcome
- An *event* is any set or collection of outcomes.
  - An event can have one outcome (simple even) or
  - More than one outcome

Examples of Events

- For drawing a card from a fair poker deck
  - Simple event: the five of spades (5♠) – this is a simple event since there is one outcome
  - Event: Five (5) – this event consists of four possible outcomes, 5♠, 5♣, 5♥, and 5♦

What is a Sample Space?

- The sample space for an experiment is the set or collection of all possible outcomes for the experiment.

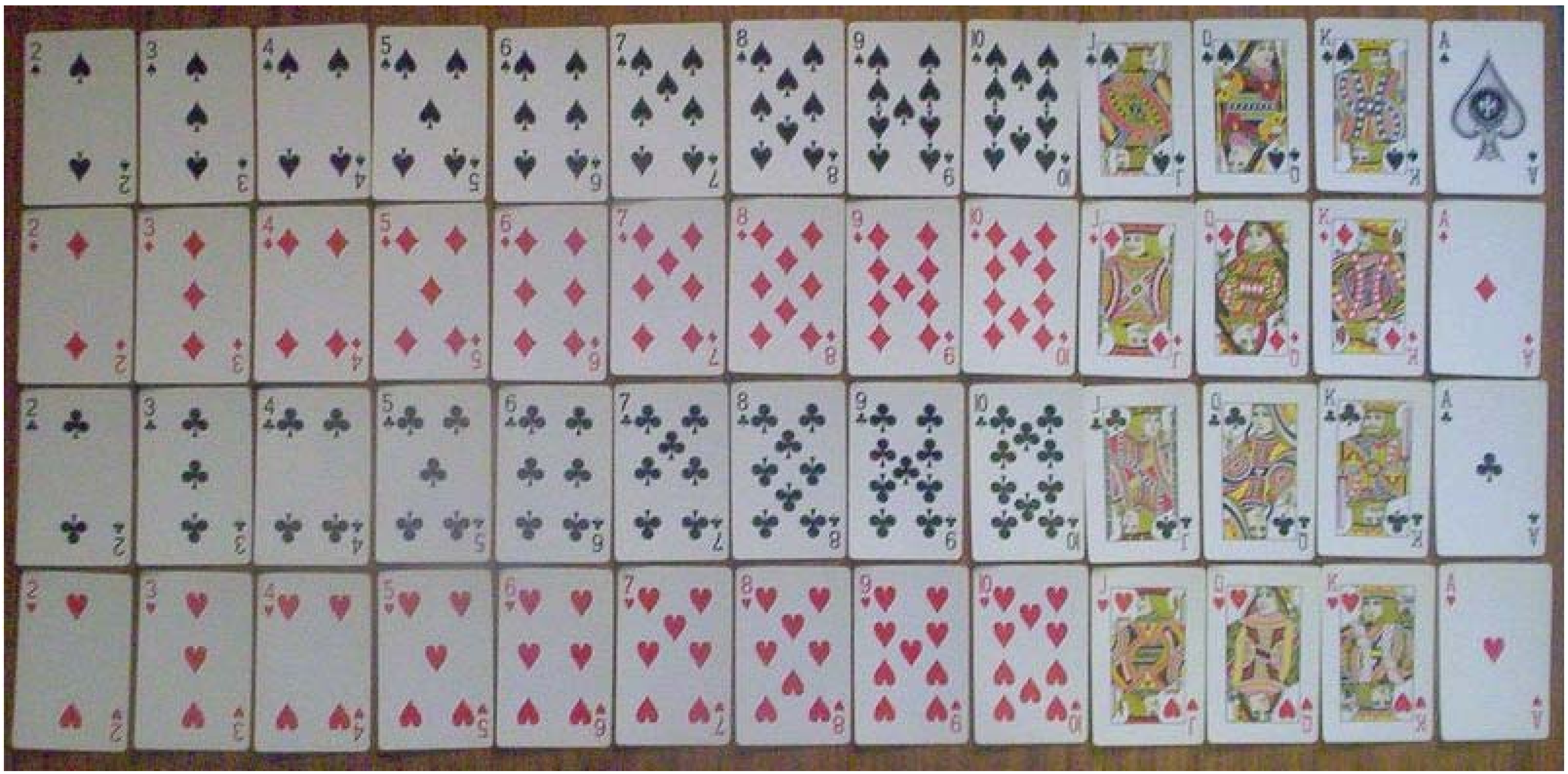
Examples of Sample Space

- The sample space for tossing a fair coin is heads and tails. That is, the two possible outcomes for tossing a fair coin are
  - The coin landing head side up – heads
  - The coin landing tail side up – tails

Examples of Sample Space

- The sample space for drawing a card from a fair poker deck is getting any one of the fifty-two (52) cards in the deck of cards.

What cards are in a fair poker deck?



What cards are in a fair poker deck?

- There are four suits
  - Spades - ♠
  - Clubs - ♣
  - Hearts - ♥
  - Diamonds - ♦
- Two suits are black - ♠, ♣
- Two suits are red - ♥, ♦

What cards are in a fair poker deck?

- Each suit has the following cards:
  - A (Ace)
  - 2 (two)
  - 3 (three)
  - 4 (four)
  - 5 (five)
  - 6 (six)
  - 7 (seven)
  - 8 (eight)
  - 9 (nine)
  - 10 (ten)
  - J (Jack)
  - Q (Queen)
  - K (King)

What cards are in a fair poker deck?

- There are thirteen (13) *kinds* of cards:
  - A (Ace)
  - 2 (two)
  - 3 (three)
  - 4 (four)
  - 5 (five)
  - 6 (six)
  - 7 (seven)
  - 8 (eight)
  - 9 (nine)
  - 10 (ten)
  - J (Jack)
  - Q (Queen)
  - K (King)

What cards are in a fair poker deck?

- For *ordering* the *kinds* of cards with *Aces Low*:
  - A (Ace)
  - 2 (two)
  - 3 (three)
  - 4 (four)
  - 5 (five)
  - 6 (six)
  - 7 (seven)
  - 8 (eight)
  - 9 (nine)
  - 10 (ten)
  - J (Jack)
  - Q (Queen)
  - K (King)

What cards are in a fair poker deck?

- For *ordering* the *kinds* of cards with *Aces High*:
  - 2 (two)
  - 3 (three)
  - 4 (four)
  - 5 (five)
  - 6 (six)
  - 7 (seven)
  - 8 (eight)
  - 9 (nine)
  - 10 (ten)
  - J (Jack)
  - Q (Queen)
  - K (King)
  - A (Ace)

What cards are in a fair poker deck?

- There is no 1 (one) in a fair poker deck.
- A face card is a card that has a face on it.
  - There are three face cards in each suit
  - J (Jack), Q (Queen), K (King)

What cards are in a fair poker deck?

- The Ace is not a face card since the Ace does not have a face on the card.
  - *Aces have no faces.*

What cards are in a fair poker deck?

- Each suit has nine (9) numbered cards:
  - 2 (two)
  - 3 (three)
  - 4 (four)
  - 5 (five)
  - 6 (six)
  - 7 (seven)
  - 8 (eight)
  - 9 (nine)
  - 10 (ten)

What cards are in a fair poker deck?

- Each suit has five (5) even numbered cards:
  - 2 (two)
  - 4 (four)
  - 6 (six)
  - 8 (eight)
  - 10 (ten)

What cards are in a fair poker deck?

- Each suit has four (4) odd numbered cards:
  - 3 (three)
  - 5 (five)
  - 7 (seven)
  - 9 (nine)

What cards are in a fair poker deck?

- Each suit has four (4) non-numbered cards:
  - A (Ace)
  - J (Jack)
  - Q (Queen)
  - K (King)

What cards are in a fair poker deck?

- Each suit has four (4) lettered cards:
  - A (Ace)
  - J (Jack)
  - Q (Queen)
  - K (King)

What cards are in a fair poker deck?

- For each suit, the lettered cards are the non-numbered cards:
  - A (Ace)
  - J (Jack)
  - Q (Queen)
  - K (King)

What cards are in a fair poker deck?

- The A (Ace) is the only non-numbered non-face card

What cards are in a fair poker deck?

- There are four of each kind of card
  - For each kind of card, there is one of each suit
    - Club, ♣
    - Spade, ♠
    - Diamond, ♦
    - Heart, ♥

What cards are in a fair poker deck?

- Since there are two (2) black suits, ♠ and ♣, half the cards in the deck are black
  - There are twenty-six (26) black cards
- Since there are two (2) red suits, ♦ and ♥, half the cards in the deck are red
  - There are twenty-six (26) red cards

What cards are in a fair poker deck?

- There are thirteen (13) cards in each suit
  - Thirteen (13) clubs, ♣
  - Thirteen (13) spades, ♠
  - Thirteen (13) diamonds, ♦
  - Thirteen (13) hearts, ♥

What cards are in a fair poker deck?

- There are
  - Twenty (20) even numbered cards
  - Sixteen (16) odd numbered cards
  - Sixteen (16) lettered cards
  - Twelve (12) face cards
  - Four (4) non-face lettered cards
  - Four (4) of each kind of card

Calculating Probability

- Imagine performing an experiment multiple times under unchanging conditions
  - Probability is the long-term proportion or relative frequency with which an event will be observed to occur

Calculating Probability

- To determine the probability of an event E:
  - Determine the number of outcomes in the event
  - Determine the number of outcomes in the sample space
  - The probability of the event is the ratio of the number of outcomes in the event and the number of events in the sample space

Calculating Probability

- Consider an experiment
  - Suppose an event E consists of m possible outcomes
  - Suppose the sample space for the experiment has n possible outcomes
  - The probability that event E occurs is m/n

Calculating Probability

- For an experiment with n possible outcomes and an event E with m possible outcomes
  - The probability that event E occurs is $P(E) = m/n$.
  - $P(E)$ denotes the probability that event E occurs
  - The fraction, m/n, must be reduced if m and n have any common factors

Properties of Probability

- For any experiment,
  - The probability that the sample space occurs is one (1): $P(S) = 1$
  - Suppose that the event E cannot occur for the experiment. The probability that an impossible event occurs is zero (0): $P(E) = 0$

Properties of Probability

- For any experiment,
  - Suppose E is any *possible* event for the experiment. The probability that event E occurs is greater then zero and less than or equal to one: $0 < P(E) \leq 1$

Properties of Probability

- For any experiment,
  - Suppose E is any event, *possible* or *impossible*, for the experiment. The probability that event E occurs is between zero and one, inclusive: $0 \leq P(E) \leq 1$

Properties of Probability

- For any experiment,
  - Suppose E_1, E_2, E_3, E_4, …, E_n, are the n possible simple events that make up the sample space, S, for the experiment: $S = \{E_1, E_2, E_3, E_4, \ldots, E_n\}$.
  - The sum of the probabilities for the simple events is one:

$$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n) = 1$$

Example

- What is the probability of drawing a red seven from a fair poker deck?

Example

- What is the probability of drawing a red seven from a fair poker deck?
  - Event has two possible outcomes since there are two red sevens.
  - The sample space has fifty-two possible outcomes since there are fifty-two cards in a fair poker deck
  - P(drawing a red seven) = 2/52
    = 1/26

Example

- What is the probability that the sum of the faces on a pair of fair dice is seven?

Example

- What is the probability that the sum of the faces on a pair of fair dice is seven?

What is the sample space for rolling a pair of fair dice?

Sample Space for Rolling a Pair of Fair Dice

- Each die displays dots.
  - The number of dots on each face of a fair die is unique
  - Each of the six faces displays one, two, three, four, five, or six dots.
  - There are thirty-six possible pairings of the two dice when each die is considered to be unique; you may consider one the first die and the other the second die

Sample Space for Rolling a Pair of Fair Dice

- In the following display, the pairing of the dice are displayed numerically within parentheses.
  - The first number represents the number of dots displayed on the top face of the first die
  - The second number represents the number of dots displayed on the top face of the second die

Sample Space for Rolling a Pair of Fair Dice

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Back to the Example

- What is the probability that the sum of the faces on a pair of fair dice is seven?

Back to the Example

- What is the probability that the sum of the faces on a pair of fair dice is seven?
  - There are six (6) possible outcomes for which the sum of the faces on the pair of fair dice is seven: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), and (6, 1)

Back to the Example

- What is the probability that the sum of the faces on a pair of fair dice is seven?
  - P(sum is seven) = 6/36
    = 1/6

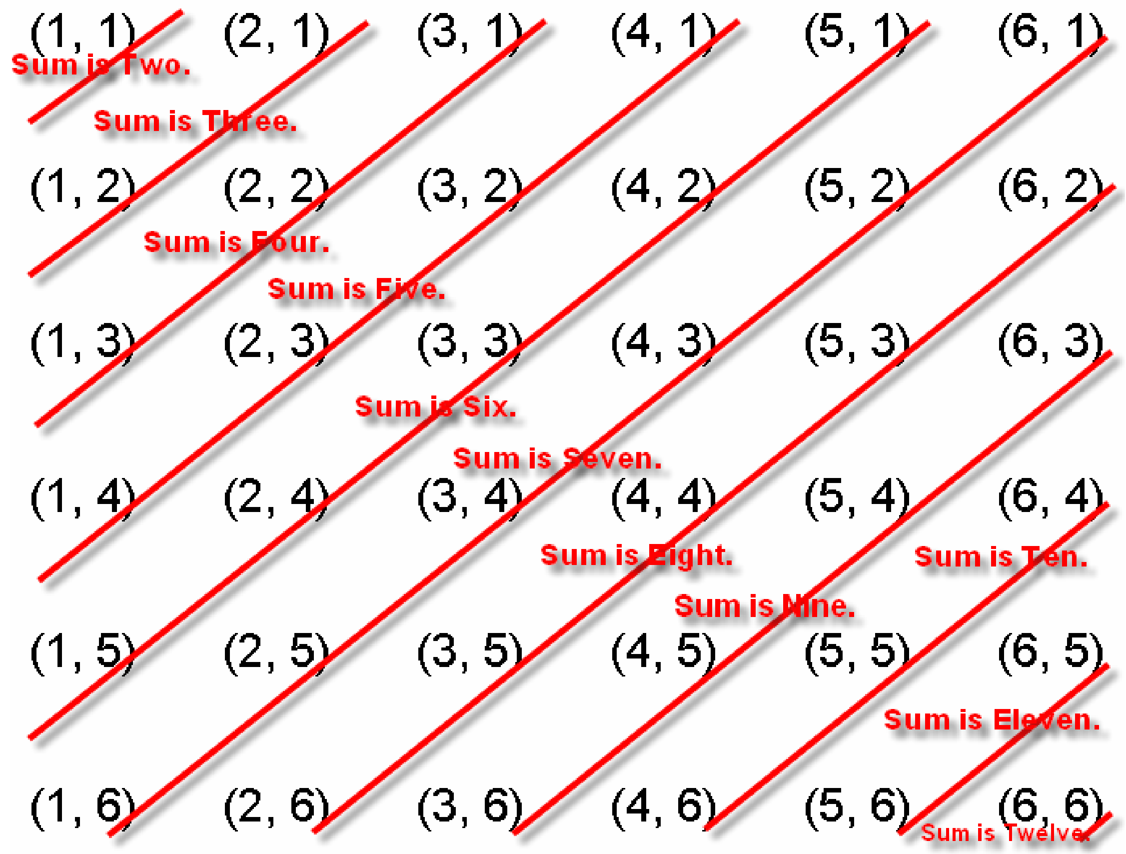
Examine the Sample Space for Rolling a Pair of Fair Dice Along the Diagonals

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

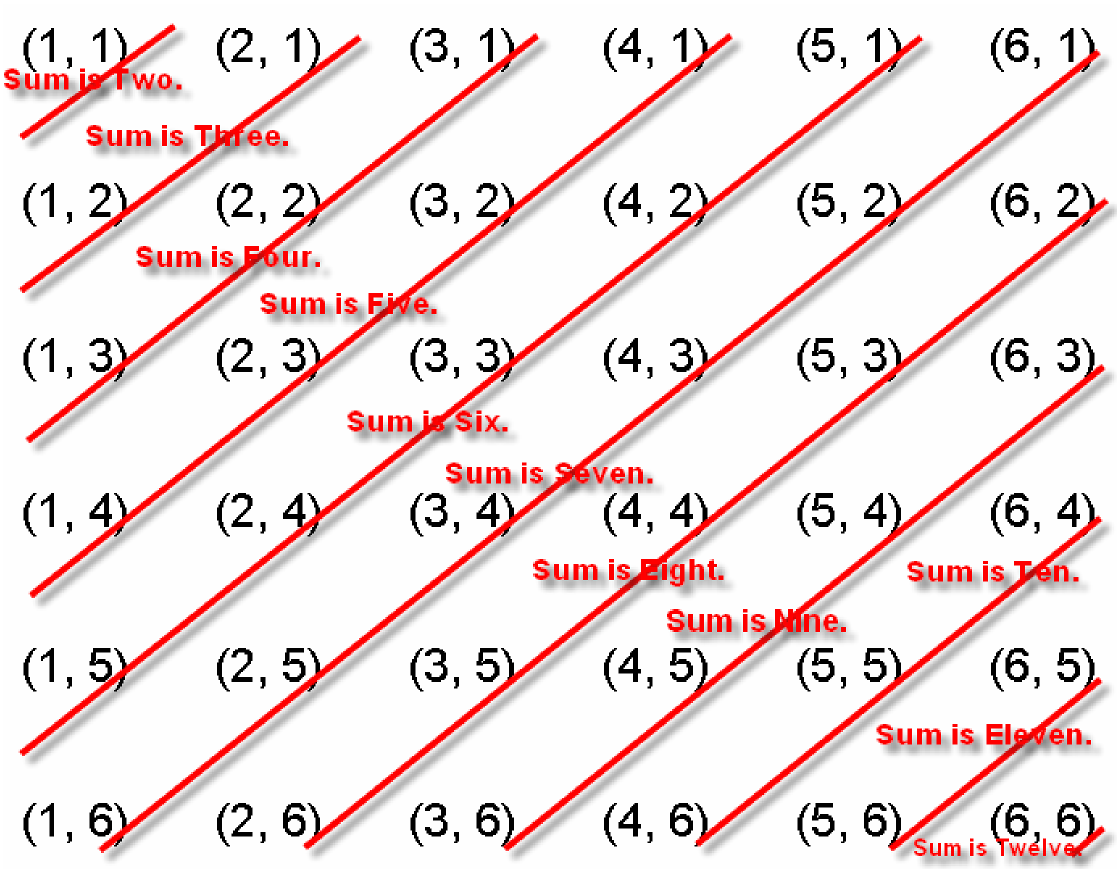
Examine the Sample Space for Rolling a Pair of Fair Dice Along the Diagonals

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Examine the Sample Space for Rolling a Pair of Fair Dice Along the Diagonals



Examine the Sample Space for Rolling a Pair of Fair Dice Along the Diagonals



We see that each diagonal has a different sum. These sums range from two (2) to twelve (12), inclusive.

What is the probability that a family with four children has two boys and two girls?

What is the probability that a family with four children has two boys and two girls?

- Based on the question, does order matter?

What is the probability that a family with four children has two boys and two girls?

- Based on the question, does order matter? No.

What is the probability that a family with four children has two boys and two girls?

- Based on the question, does order in which the children are born matter? No.
- Why?

What is the probability that a family with four children has two boys and two girls?

- Based on the question, does order in which the children are born matter? No.
- Why? The birth order does not matter since the question does not tell us that the order does matter. If the question included order restrictions then we would use the restrictions in our determination of the sample space and the event.

What is the probability that a family with four children has two boys and two girls?

- To determine this probability, we must know the sample space.

What is the probability that a family with four children has two boys and two girls?

- How do we determine sample space for a four-child family?

What is the probability that a family with four children has two boys and two girls?

- How do we determine sample space for a four-child family?
  - Write out all the possible ways in which each child could be a boy, B, or a girl, G.

What is the probability that a family with four children has two boys and two girls?

- How many possibilities are there?

What is the probability that a family with four children has two boys and two girls?

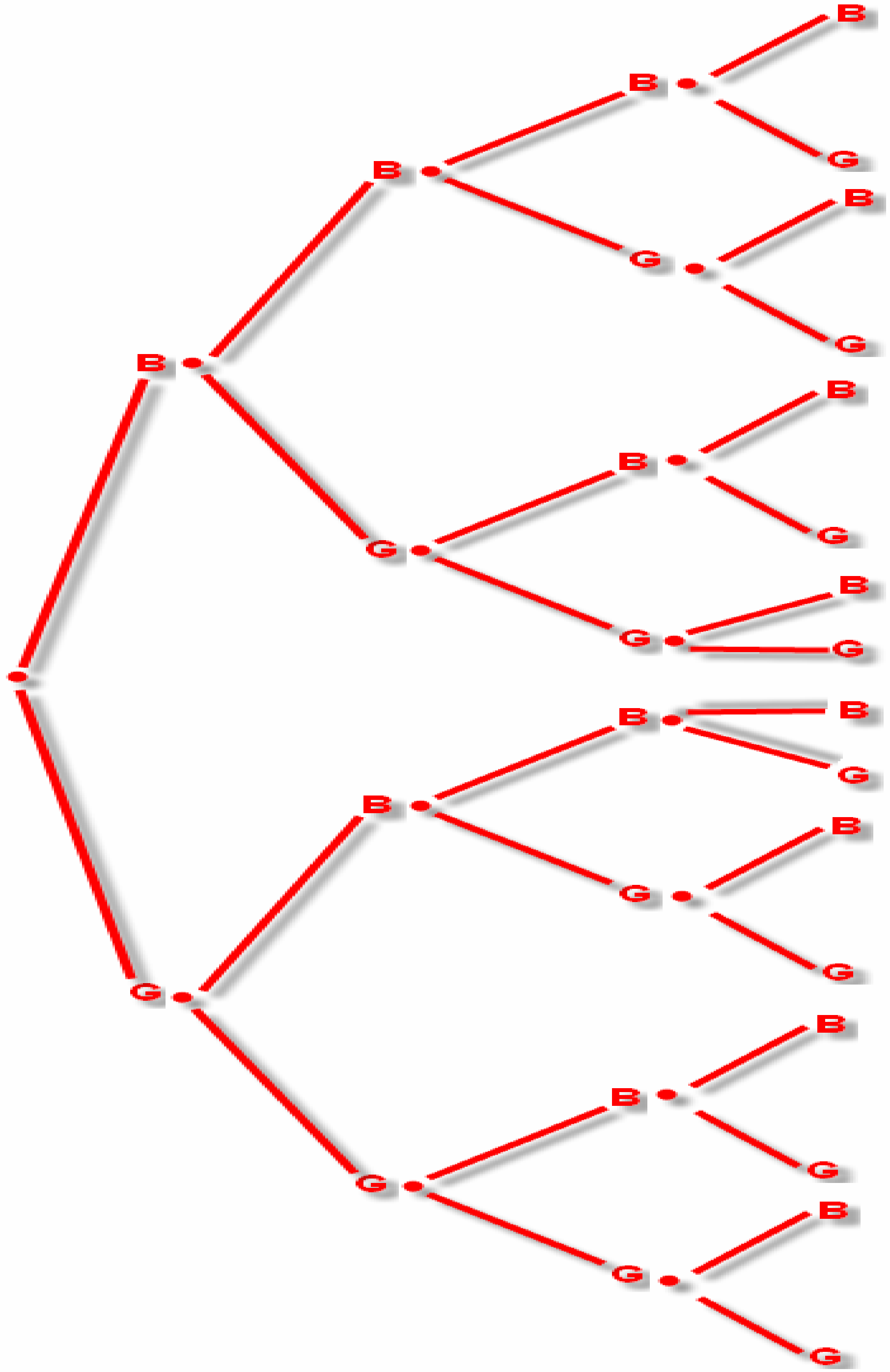
- How many possibilities are there?
  - For each child, first, second, third, and fourth, there are two possibilities, boy or girl.
  - There are $2(2)(2)(2) = 2^4 = 16$ possibilities.

What is the probability that a family with four children has two boys and two girls?

- What are all these possibilities?

What is the probability that a family with four children has two boys and two girls?

- What are all these possibilities?
  - You will find it easier to determine this sample space if you create a tree diagram.
    - First Branch: Put in the possibilities for the first child, B or G
    - Second Branch: For each of these put in the possibilities for the second child, B or G
    - Continue in this manner until you have all four branches, one for each child



Sample Space for a Four-Child Family

Sample Space for a Four-Child Family

- BBBB
- BBBG
- BBGB
- BBGG
- BGBB
- BGBG
- BGGB
- BGGG
- GBBB
- GBBG
- GBGB
- GBGG
- GGBB
- GGBG
- GGGB
- GGGG

What is the probability that a family with four children has two boys and two girls?

- How many four-child families have two boys and two girls?

Four-Child Families with two boys and two girls

- BBGG
- BGBG
- BGGB
- GBBG
- GBGB
- GGBB

What is the probability that a family with four children has two boys and two girls?

- How many four-child families have two boys and two girls? Six

What is the probability that a family with four children has two boys and two girls?

- How many four-child families have two boys and two girls? Six
- How many four-child families are possible? That is, how many simple events are in the sample space?

What is the probability that a family with four children has two boys and two girls?

- How many four-child families have two boys and two girls? Six
- How many four-child families are possible? That is, how many simple events are in the sample space? Sixteen

What is the probability that a family with four children has two boys and two girls?

- How many four-child families have two boys and two girls? Six
- How many four-child families are possible? Sixteen
- What is the probability that a four-child family has two boys and two girls?

What is the probability that a family with four children has two boys and two girls?

- How many four-child families have two boys and two girls? Six
- How many four-child families are possible? Sixteen
- What is the probability that a four-child family has two boys and two girls? $P(2B, 2G) = 6/16$

  $= 3/8$

What must we know in order to determine probability?

- Sample space - all possible simple events for the experiment
  - Number of simple events in the sample space
- Event – all simple events in the event
  - Number of simple events in the event