Hypothesis Testing

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- What are the null and alternative hypotheses?
- What are the errors made in hypothesis testing?
- How do we test a hypothesis?
- How do we state conclusions for hypothesis tests?

What is a Hypothesis?

- A hypothesis is
  - a proposal intended to explain certain facts or observations;
  - an assumption taken to be true for the purpose of argument or investigation;
  - a tentative statement or supposition which may be tested through research.

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- A hypothesis is a statement/claim regarding a ***parameter*** of one or more populations.
- The parameter could be the population mean or a population proportion, for example.

Two Hypotheses to Consider …

- Null Hypothesis
- Alternative Hypothesis or Research Hypothesis

What is the Null Hypothesis?

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What is the Null Hypothesis?

- The null hypothesis is
  - the accepted standard;
  - the status quo hypothesis that has been assumed to be true

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- The null hypothesis is a statement claiming that there is no change, no difference, no effect.

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- The null hypothesis is a statement/claim that is tentatively assumed to be true regarding the value of a parameter for a population.

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- **The alternative hypothesis, denoted H_1 and read "H-one", is a statement/claim to be tested for which *we are trying to find evidence*.**

What is the Alternative Hypothesis?

- **The alternative hypothesis is a statement that *might* be true instead of the null hypothesis.**

What is the Alternative Hypothesis?

- The alternative hypothesis is an alternative statement/claim that is being tested.

What is the Alternative Hypothesis?

- The alternative hypothesis is an alternative statement/claim regarding the value of a parameter for a population.

What is a Hypothesis Test?

- A hypothesis test, also known as a test of significance, is a procedure that compares the results from a sample to some predetermined standard in order to decide whether the standard should/can be rejected.

What is a Hypothesis Test?

- A hypothesis test is a procedure that uses information obtained from a sample to determined if the null hypothesis should/can be rejected.

What is a Hypothesis Test?

- *A* hypothesis test is a procedure that uses information obtained from a sample to determined if the null hypothesis should/can be *rejected and* the alternative hypothesis *accepted*.

What is Hypothesis Testing?

- Hypothesis testing the procedure for choosing between the null hypothesis H_0 and the alternative hypothesis H_1.

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- Hypothesis testing a process used to test claims regarding a ***parameter*** of one or more populations based on sample evidence and probability.

Hypothesis Testing????

- The null hypothesis H_0 will be rejected and the alternative hypothesis H_1 accepted only if the sample data suggests beyond a reasonable doubt that the null hypothesis is not true.

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- The null hypothesis H_0 will be rejected and the alternative hypothesis H_1 accepted only if the sample data suggests ***beyond a reasonable doubt*** that the null hypothesis is not true.

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- So, consider yourself as a member of a jury: you must *assume* that the defendant is *innocent until* **proven guilty**.
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- So, consider yourself as a member of a jury: you must *assume* that the defendant is *innocent until* **proven guilty**.
  - H_0: The defendant is innocent.
  - H_1: The defendant is *guilty*.
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Typical Null Hypothesis

- H_0: parameter = some value

where "parameter" is a parameter for the population.

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Note: Professional statisticians and professional journals use only the equal symbol for equality; it is rare that $\leq$ or $\geq$ are used. Hypothesis tests are conducted by assuming that the parameter is equal to some value.

Alternative Hypothesis

- Three possibilities
  - H_1: parameter < some value
  - H_1: parameter > some value
  - H_1: parameter ≠ some value

where "parameter" is a parameter for the population.

Pairings of H_0 and H_1

- Pairing the null hypothesis H_0 with the three possibilities H_1

  we obtain three tests …

Left-Tailed Test

- H_0: parameter = some value
  H_1: parameter < some value

where "parameter" is a parameter for the population.

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  H_1: parameter < some value

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Note: "<" points to the left and, thus, the name left-tailed test.

Right-Tailed Test

- H_0: parameter = some value
  H_1: parameter > some value

where "parameter" is a parameter for the population.

Right-Tailed Test

- H_0: parameter = some value
  H_1: parameter > some value

where "parameter" is a parameter for the population.

Note: ">" points to the right and, thus, the name right-tailed test.

One-Tailed Tests

The left-tailed test,

- H_0: parameter = some value
  H_1: parameter < some value,

and the right-tailed test,

- H_0: parameter = some value
  H_1: parameter > some value,

are collectively referred to as one-tailed tests.

Two-Tailed Test

- H_0: parameter = some value
  H_1: parameter $\neq$ some value

where "parameter" is a parameter for the population.

Two-Tailed Test

- H_0: parameter = some value
  H_1: parameter $\neq$ some value

where "parameter" is a parameter for the population.

Note: If the parameter is not equal to the value then the parameter could be greater than or less than the value - two directions.

Recall

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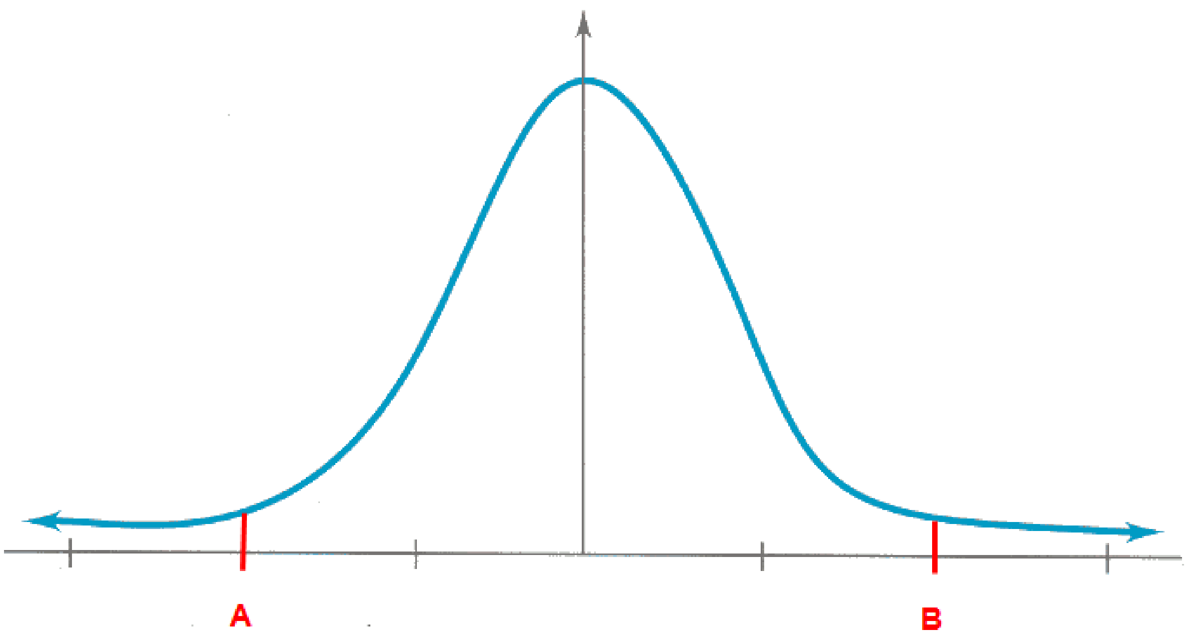
  - For example, we obtain a sample mean and determine the interval estimates together with the level of confidence for the population mean, that is, the interval that contains the population mean with a certain level of confidence.

Why "Tails"?

- Think about the confidence interval – with a certain level of confidence, we know that the parameter is in the interval from A to B.



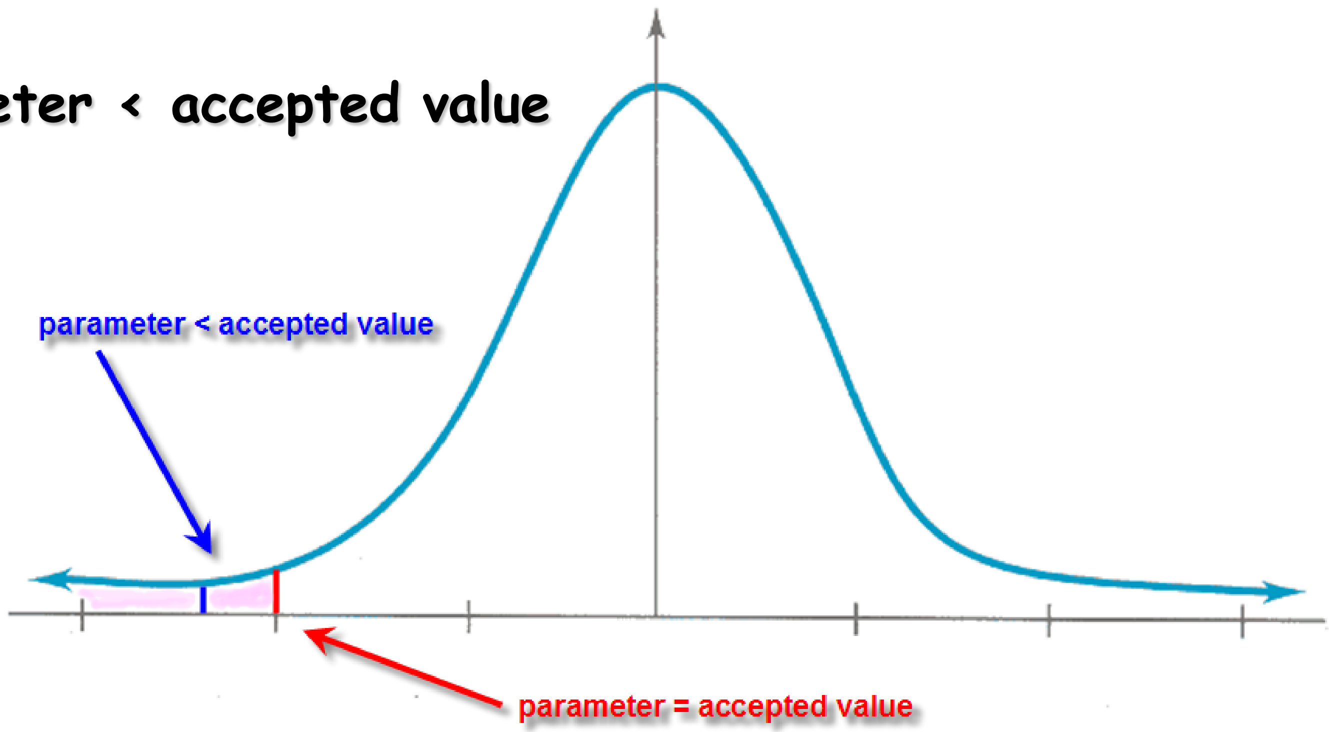
Why "Tails"?

- We could have

  parameter = accepted value

  or

  parameter < accepted value



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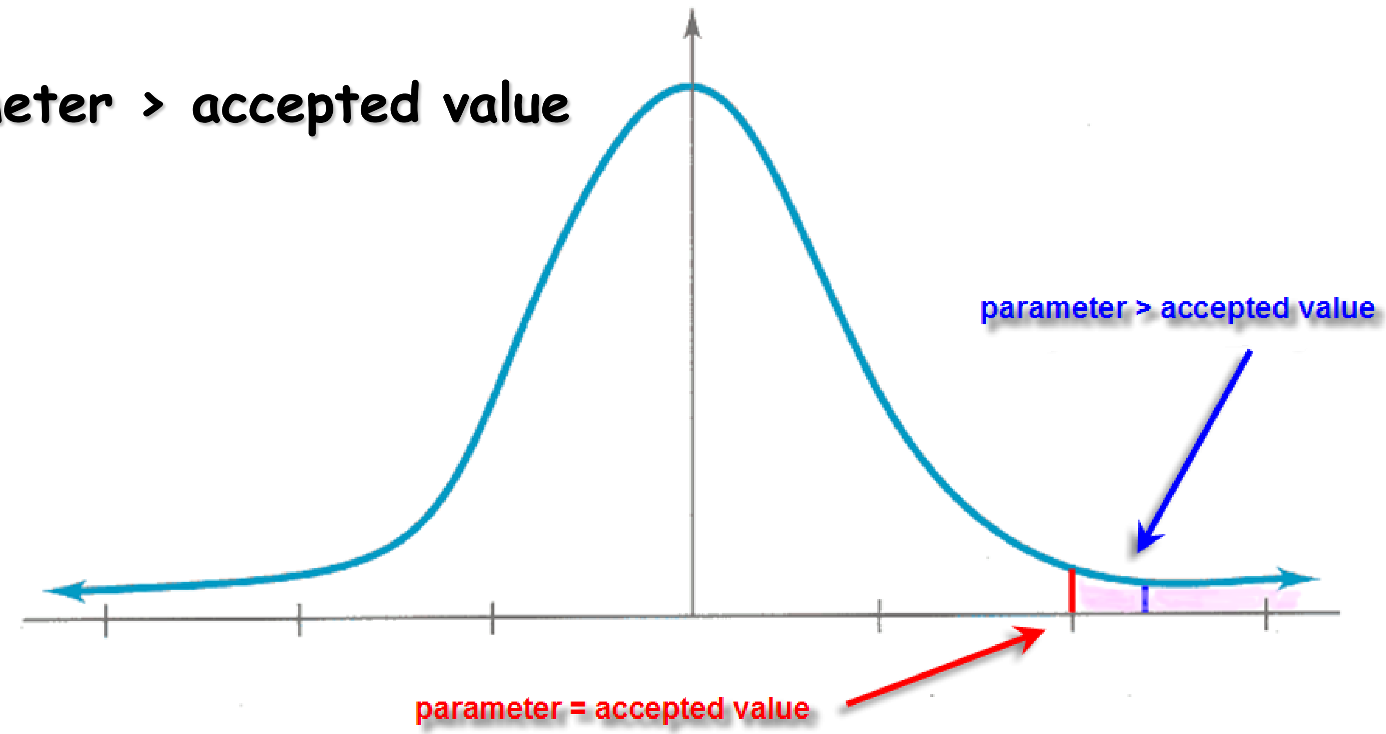

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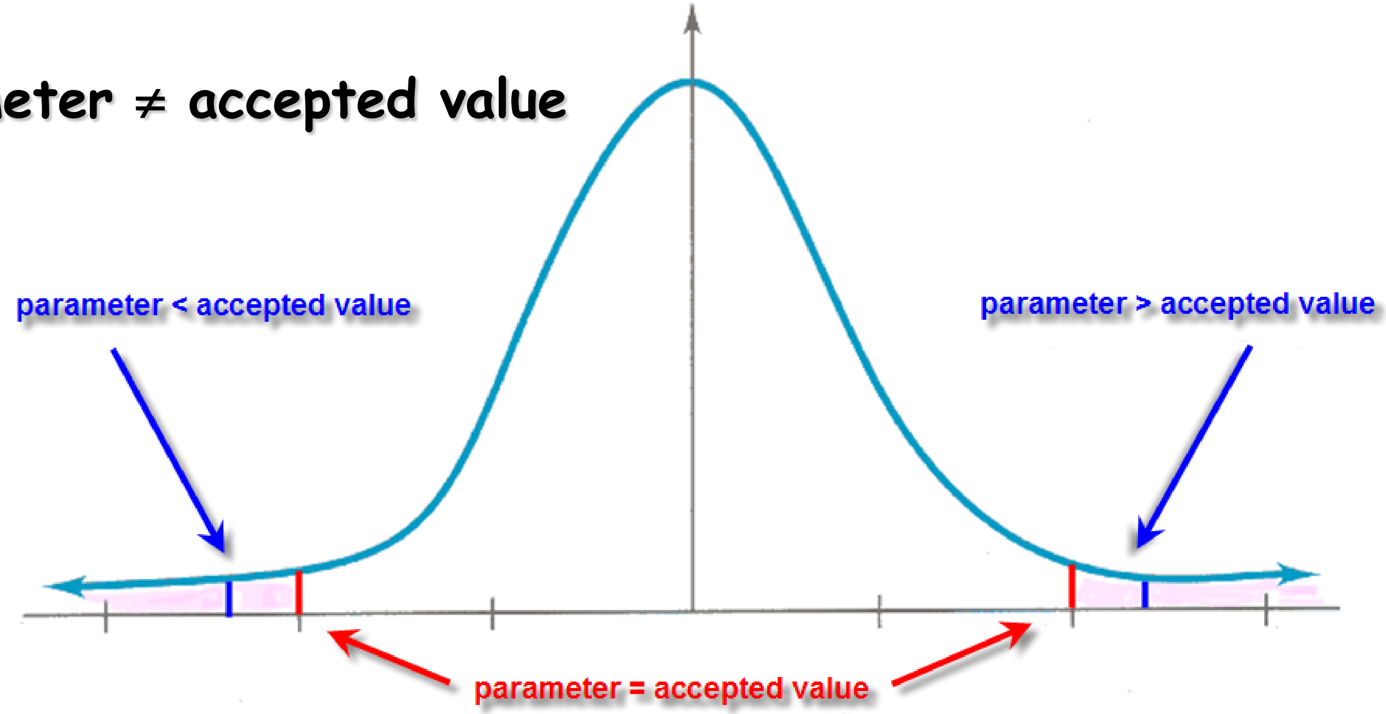

Tails?

- The “tails” of interest are the regions in the standard normal distribution and t-distribution where the data tails-off.

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Tails?

- Before we can continue to explore the idea of "tails" and how to use them in hypothesis testing, we must understand how to determine H_0 and H_1 as well as the **errors** that may occur.

How do we set up H_0 and H_1?

- Consider the following example …

How do we set up H_0 and H_1?

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  - Claim: $\mu > 800$

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  - Claim: $\mu > 800$
  - This claim is the alternative hypothesis H_1.

How do we set up H_0 and H_1?

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average.
  - Create a statement involving μ, an equal sign, and 800.

How do we set up H_0 and H_1?

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average.
  - Statement: $\mu = 800$

How do we set up H_0 and H_1?

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average.
  - Statement: $\mu = 800$
  - This is a statement of no difference, no change, no effect.

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  - Statement: $\mu = 800$
  - This statement is the null hypothesis H_0

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  - H_0: $\mu = 800$
  - H_1: $\mu > 800$

How do we set up H_0 and H_1?

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average.
  - $H_0: \mu = 800$
  - $H_1: \mu > 800$

Note: We do a right-tailed test.

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- According to fueleconomy.gov, Volkswagen claims that their diesel vehicles have an average gas mileage of 29.5 miles per gallon in city driving.
  - Claim: $\mu = 29.5$

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- According to fueleconomy.gov, Volkswagen claims that their diesel vehicles have an average gas mileage of 29.5 miles per gallon in city driving.
  - Claim: $\mu = 29.5$
  - This is the null hypothesis H_0 since the statement has no change/difference *and* has an equal sign.

How do we set up H_0 and H_1?

- According to fueleconomy.gov, Volkswagen claims that their diesel vehicles have an average gas mileage of 29.5 miles per gallon in city driving.
  - What is H_1?

How do we set up H_0 and H_1?

- According to fueleconomy.gov, Volkswagen claims that their diesel vehicles have an average gas mileage of 29.5 miles per gallon in city driving.
  - Since the statement provides no information that would lead us to believe that $\mu < 29.5$ or $\mu > 29.5$, we use $\mu \neq 29.5$ as the alternative hypothesis H_1.

How do we set up H_0 and H_1?

- According to fueleconomy.gov, Volkswagen claims that their diesel vehicles have an average gas mileage of 29.5 miles per gallon in city driving.
  - H_0: $\mu = 29.5$
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- According to fueleconomy.gov, Volkswagen claims that their diesel vehicles have an average gas mileage of 29.5 miles per gallon in city driving.
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Note: We do a two-tailed test.

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- According to the United States Department of Education, of those who graduated from high school from October 2005 to October 2006, 66% were attending college in October 2006. However, according to the Bureau of Labor Statistics, the percentage is lower.

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- According to the United States Department of Education, of those who graduated from high school from October 2005 to October 2006, 66% were attending college in October 2006. However, according to the Bureau of Labor Statistics, the percentage is lower.
  - First statement: $p = 0.66$

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- According to the United States Department of Education, of those who graduated from high school from October 2005 to October 2006, 66% were attending college in October 2006. However, according to the Bureau of Labor Statistics, the percentage is lower.
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  - Second statement: $p < 0.66$
  - This statement is the alternative hypothesis H_1.

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- According to the United States Department of Education, of those who graduated from high school from October 2005 to October 2006, 66% were attending college in October 2006. However, according to the Bureau of Labor Statistics, the percentage is lower.
  - H_0: $p = 0.66$
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- According to the United States Department of Education, of those who graduated from high school from October 2005 to October 2006, 66% were attending college in October 2006. However, according to the Bureau of Labor Statistics, the percentage is lower.
  - H_0: $p = 0.66$
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Note: We do a left-tailed test.

Errors???

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Errors???

- We use sample data to determine whether to reject or not reject the null hypothesis H_0.
- Using sample data,
  - our information is incomplete and
  - the use of incomplete information could lead to our making an incorrect decision.

Possible Outcomes from Hypothesis Testing

- Reject H_0 when H_1 is true: this decision would be correct.
- Do not reject H_0 when H_0 is true: this decision would be correct.
- Reject H_0 when H_0 is true: this decision would be incorrect. This is called a Type I error.
- Do not reject H_0 when H_1 is true: this decision would be incorrect. This is called a Type II error.

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Possible Outcomes from Hypothesis Testing

		Reality	
		H_0 is True	H_1 is True
Conclusion	Do not Reject H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

Possible Outcomes from Hypothesis Testing

		Reality	
		H_0 is True	H_1 is True
Conclusion	Do not Reject H_0	Correct Conclusion	*Type II Error*
	Reject H_0	*Type I Error*	Correct Conclusion

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- We make a Type I error if we believe that $\mu > 800$ (reject the null hypothesis) when the average amount of sodium in one serving of ready-to-eat or add-water soup is 800 mg (null hypothesis is actually true).

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- We make a Type I error if we *believe* that the average gas mileage for Volkswagen diesel vehicles *is not 29.5 miles per gallon* (reject the null hypothesis) when the average gas mileage *is 29.5 miles per gallon* (null hypothesis is actually true).

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- **We make a Type II error if we believe that *the average gas mileage for Volkswagen diesel vehicles is 29.5 miles per gallon* when the average gas mileage *is actually not 29.5 miles per gallon.***

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- We make a Type I error if we *believe* that the percentage of students who graduated from high school from October 2005 to October 2006 that were attending college in October 2006 *is less than 66%* (reject the null hypothesis) when the percentage *is 66%* (the null hypothesis is actually true).

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- We make a Type II error if we do not reject the null hypothesis when the alternative hypothesis is actually true.

What would it mean to make a Type I Error or a Type II Error?

- According to the United States Department of Education, of those who graduated from high school from October 2005 to October 2006, 66% were attending college in October 2006. However, according to the Bureau of Labor Statistics, the percentage is lower.
  - Recall - H_0: $p = 0.66$
    H_1: $p < 0.66$
- We make a Type II error if we believe that the percentage of students who graduated from high school from October 2005 to October 2006 that were attending college in October 2006 *is 66%* (do not reject the null hypothesis) when the percentage *is less than 66%* (alternative hypothesis is actually true).

What would it mean to make a Type I Error or a Type II Error?

- According to the United States Department of Education, of those who graduated from high school from October 2005 to October 2006, 66% were attending college in October 2006. However, according to the Bureau of Labor Statistics, the percentage is lower.
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    H_1: $p < 0.66$
- We make a Type II error if we believe that the percentage of students who graduated from high school from October 2005 to October 2006 that were attending college in October 2006 *is* 66% when the percentage *is less than* 66%.

Level of Significance

- The level of significance, α, is the probability of making a Type I error.
  - P(Type I error)
    = P(rejecting H_0 when H_0 is true)
    = α

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- Level of significance depends on the consequences of making a Type I error.

Level of Significance

- The level of significance, α, is the probability of making a Type I error.
  - P(Type I error) = α
- Level of significance depends on the consequences of making a Type I error.
- If consequences
  - severe then take $\alpha = 0.01$
  - Not severe then take $\alpha = 0.05$
    or $\alpha = 0.10$

Level of Significance

- The level of significance, α, is the probability of making a Type I error.
  - P(Type I error)
    = P(rejecting H_0 when H_0 is true)
    = α

Note:
P(Type II error)
= P(not rejecting H_0 when H_1 is true)
= β

Level of Significance

- CAUTION: Reducing the probability of making a Type I error, α, increases the probability of making a Type II error, β

Testing Hypothesis about μ with *Unknown* σ

- We use the t-distribution

Testing Hypothesis about μ with *Unknown* σ

- Requirements:
  - Determine the null (H_0: $\mu = \mu_0$) and alternative hypothesis
    - Determine if left-tailed, right-tailed, or two-tailed

Testing Hypothesis about μ with *Unknown* σ

- Requirements:
  - Obtain a simple random sample (with no outliers) from the population
  - Population must be normal or the sample size must be large ($n \geq 30$)
  - Use test statistic $$t_0 = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$$

    and Student's t-distribution with $n - 1$ degrees of freedom

Testing Hypothesis about μ with *Known* σ

- We use the standard normal table

Testing Hypothesis about μ with *Known* σ

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  - Determine the null (H_0: $\mu = \mu_0$) and alternative hypothesis
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Testing Hypothesis about μ with *Known* σ

- Requirements:
  - Obtain a simple random sample (with no outliers) from the population
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$$z_0 = \frac{\bar{x} - \mu_0}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Testing Hypothesis about a Proportion p

- We use the standard normal table

Testing Hypothesis about a Proportion p

- Requirements:
  - Determine the null (H_0: $p = p_0$) and alternative hypothesis
    - Determine if left-tailed, right-tailed, or two-tailed

Testing Hypothesis about a Proportion p

- Requirements:
  - Obtain a simple random sample from the population
  - Meet conditions: $np_0(1 - p_0) \geq 10$ with $n \leq 0.05N$
  - Use test statistic

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0\left(1 - p_0\right)}{n}}}$$

Two Approaches

- Classical
- p-Value Approach

Two Approaches

- Classical
- p-Value Approach

Note: We could use a confidence interval approach as well.

Two Approaches

- Classical
  - Compare the test statistic with the critical value determined by the level of significance
- p-Value Approach

Two Approaches

- Classical
  - Compare the test statistic with the critical value determined by the level of significance
- p-Value Approach
  - Compare the p-value (probability associated with test statistic) to the level of significance

Two Approaches

- Classical
- p-Value Approach

We will explore these methods in the context of two examples.

Two Approaches

- Classical
- p-Value Approach

We will explore these methods in the context of two examples, the first for the population mean, μ, and the second for the population proportion, p.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

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- Null hypothesis $H_0: \mu = 800$
  Alternative hypothesis $H_1: \mu > 800$

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Null hypothesis H_0: $\mu = 800$
  Alternative hypothesis H_1: $\mu > 800$

- We will use a right-tailed test

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- We take μ_0 = 800, n = 31, ˜ = 845.7, and s = 168.3

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.
- We use these (μ_0 = 800, n = 31, ˜ = 845.7, and s = 168.3) to determine the value of the test statistic.

$$t_0 = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$$

- **The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.**

$$t_0 = \frac{845.7 - 800}{\left(\frac{168.3}{\sqrt{31}}\right)}$$

$$= 1.511864714$$

- **The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.**

$$t_0 = \frac{845.7 - 800}{\left(\frac{168.3}{\sqrt{31}}\right)}$$

$$\approx 1.512$$

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$$t_0 = \frac{845.7 - 800}{\left(\frac{168.3}{\sqrt{31}}\right)}$$

$$\approx 1.512$$

We take three decimal places since critical values in the t-distribution table are recorded to three decimal places.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

$$t_0 = \frac{845.7 - 800}{\left(\frac{168.3}{\sqrt{31}}\right)} \approx 1.512$$

There are 30 degrees of freedom since

$$n - 1 = 31 - 1 = 30.$$

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Level of significance

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Let us consider a 0.10 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- For a 0.10 level of significance, the probability that a Type I error occurs is 0.1.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Using the information we have gathered, we will determine if we can reject the null hypothesis at the 0.10 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method I: Classical Approach

Classical Approach for Hypothesis testing for μ with σ Unknown

- Determine the critical value.

Classical Approach for Hypothesis testing for μ with σ Unknown

- Determine the critical value in the t-distribution table corresponding to the level of significance α with n – 1 degrees of freedom.
  - Critical values
    - Left-tailed: $-t_\alpha$
    - Right-tailed: t_α
    - Two-tailed: $-\ t_{\alpha/2}$ and $t_{\alpha/2}$

Classical Approach for Hypothesis testing for μ with σ Unknown

- Determine the critical value in the t-distribution table corresponding to the level of significance α with n – 1 degrees of freedom.
- Compare the critical value, t_α, to the test statistic, t_0.

Classical Approach for Hypothesis testing for μ with σ Unknown

- Compare the critical value to the test statistic, t_0.
  - Reject null hypothesis H_0
    - Left-tailed: if $t_0 < -t_\alpha$
    - Right-tailed: if $t_0 > t_\alpha$
    - Two-tailed: if $t_0 < -t_{\alpha/2}$ or $t_0 > t_{\alpha/2}$

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method I: Classical Approach
    - We determine the critical value, t_α, in the t-distribution table that corresponds to $\alpha = 0.10$ with 30 degrees of freedom.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method I: Classical Approach
  - Compare $t_\alpha = 1.310$ and $t_0 = 1.512$

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method I: Classical Approach
  - Compare $t_0 = 1.512$ and $t_\alpha = 1.310$
  - Since $t_0 > t_\alpha$, we can reject the null hypothesis at the 0.10 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

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- Method II: p-Value Approach

p-Value Approach for Hypothesis testing for μ with σ Unknown

- Determine the p-value.

p-Value Approach for Hypothesis testing for μ with σ Unknown

- Determine the p-value.
  - p-Value is the area
    - Left-tailed: $P(t < t_0)$
    - Right-tailed: $P(t > t_0)$
    - Two-tailed:

      $P(t < -|t_0| \text{ or } t > |t_0|)$

p-Value Approach for Hypothesis testing for μ with σ Unknown

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  - p-Value is the area
    - Left-tailed: $P(t < t_0)$
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$$P(\, t < -|t_0|) + P(t > |t_0|)$$

p-Value Approach for Hypothesis testing for μ with σ Unknown

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    - Left-tailed: $P(t < t_0)$
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$$P(t < -|t_0|) + P(t > |t_0|)$$

- Compare p-value to α

p-Value Approach for Hypothesis testing for μ with σ Unknown

- Determine the p-value.
  - p-Value is the area
    - Left-tailed: $P(t < t_0)$
    - Right-tailed: $P(t > t_0)$
    - Two-tailed:

$$P(\, t < -|t_0|) + P(t > |t_0|)$$

- Reject the null hypothesis if p-value < α

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method II: p-Value Approach
- Examining the t-distribution table for 30 degrees of freedom, we do not find $t_0 = 1.512$.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method II: p-Value Approach
- Since $1.310 < 1.512 < 1.697$, we know that the corresponding p-value is such that

$$0.05 < \text{p-value} < 0.10$$

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method II: p-Value Approach
- Since p-value < 0.10, we can reject the null hypothesis at the 0.10 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Method I: Classical Approach
  - Since $t_0 > t_\alpha$, we can reject the null hypothesis at the 0.10 level of significance.
- Method II: p-Value Approach
  - Since p-value < 0.10, we can reject the null hypothesis at the 0.10 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Conclusion:
  - We can reject the null hypothesis at the 0.10 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Conclusion:
  - At the 0.10 level of significance, there is sufficient evidence to conclude that ready-to-eat or add-water soups have, on average, more than 800 mg of sodium.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- What if the level of significance is 0.05?

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Classical Approach with $\alpha = 0.05$:
  - Compare $t_0 = 1.512$ and $t_\alpha = 1.697$.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Classical Approach with $\alpha = 0.05$:
  - Comparing $t_0 = 1.512$ and $t_\alpha = 1.697$, we see that $t_0 < t_\alpha$.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Classical Approach with $\alpha = 0.05$:
  - Since t_0 is not greater than t_α, we cannot reject the null hypothesis at the 0.05 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- p-value Approach with $\alpha = 0.05$:
  - Since $1.310 < 1.512 < 1.697$, we know that the corresponding p-value is such that

$$0.05 < \text{p-value} < 0.10$$

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- p-value Approach with $\alpha = 0.05$:
  - Since p-value > 0.05, we cannot reject the null hypothesis at the 0.05 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Classical Approach with $\alpha = 0.05$:
  - Since t_0 is not greater than t_α, we cannot reject the null hypothesis.
- p-value Approach with $\alpha = 0.05$:
  - Since p-value > 0.05, we cannot reject the null hypothesis.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Conclusion:
  - We cannot reject the null hypothesis at the 0.05 level of significance.

- The recommended daily allowance of sodium is 2400 mg. A nutritionist claims that one serving of ready-to-eat or add-water soup contains more than 800 mg of sodium, on average. For her sample of 31 ready-to-eat or add-water soups, the average amount of sodium in one serving was 845.7 mg with a standard deviation of 168.3 mg.

- Conclusion:
  - At the 0.05 level of significance, there is not sufficient evidence to conclude that ready-to-eat or add-water soups have, on average, more than 800 mg of sodium.

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- Null hypothesis H_0: $p = 0.72$
  Alternative hypothesis H_1: $p > 0.72$

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- Null hypothesis H_0: p = 0.72
  Alternative hypothesis H_1: p > 0.72
- **We use a right-tailed test.**

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **The *Fox News* poll serves as our sample.**

- In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?

- We take p_0 = 0.72, ê= 0.75, and n = 900.

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Using $p_0 = 0.72$ and $n = 900$, we determine the value of $np_0(1 - p_0)$.**

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- $$\begin{aligned} np_0(1 - p_0) &= 900(0.72)(1 - 0.72) \\ &= 900(0.72)(0.28) \\ &= 181.44 \\ &\geq 10 \end{aligned}$$

- In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?

- Since n = 900 is less than 5% of the population, $n \le 0.05N$.

- In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?

- We use these (p_0 = 0.72, ê= 0.75, and n = 900) to determine the value of the test statistic.

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0\left(1 - p_0\right)}{n}}}$$

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

$$z_0 = \frac{0.75 - 0.72}{\sqrt{\frac{0.72(1-0.72)}{900}}}$$

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$$z_0 = \frac{0.75 - 0.72}{\sqrt{\frac{0.72(0.28)}{900}}}$$

$$\approx 2.004459314$$

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$$z_0 = \frac{0.75 - 0.72}{\sqrt{\frac{0.72(0.28)}{900}}} \approx 2.00$$

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$$z_0 = \frac{0.75 - 0.72}{\sqrt{\frac{0.72(0.28)}{900}}} \approx 2.00$$

We use two decimal places since the z-scores on the standard normal table are recorded to two decimal places.

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Let us consider the 0.05 level of significance.**

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- Let us consider the 0.05 level of significance.
  - $\alpha = 0.05$

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method I: Classical Approach**

Classical Approach for Hypothesis Testing for Proportion p

- Determine the critical value.

Classical Approach for Hypothesis Testing for Proportion p

- Determine the critical value in the standard normal table corresponding to the level of significance α.
  - Critical values
    - Left-tailed: $-z_\alpha$
    - Right-tailed: z_α
    - Two-tailed: $-z_{\alpha/2}$ and $z_{\alpha/2}$

Classical Approach for Hypothesis Testing for Proportion p

- Critical value for level of significance α.
  - Left-tailed:
    $-z_\alpha$ is such that $P(z < -z_\alpha) = \alpha$
  - Right-tailed:
    z_α is such that $P(z > z_\alpha) = \alpha$
  - Two-tailed: $-z_{\alpha/2}$ and $z_{\alpha/2}$ are such that $P(z < -z_{\alpha/2}) + P(z > z_{\alpha/2}) = \alpha$

Classical Approach for Hypothesis Testing for Proportion p

- Determine the critical value in the standard normal table corresponding to the level of significance α.
- Compare the critical value to the test statistic, z_0.

Classical Approach for Hypothesis Testing for Proportion p

- Compare the critical value to the test statistic, z_0.
  - Reject null hypothesis H_0
    - Left-tailed: if $z_0 < -z_\alpha$
    - Right-tailed: if $z_0 > z_\alpha$
    - Two-tailed: if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method I: Classical Approach**
  - **For $\alpha = 0.05$, $z_\alpha = 1.645$.**

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method I: Classical Approach**
  - **For $\alpha = 0.05$, $z_\alpha = 1.645$.**
  - ***We take the average of 1.64 and 1.65 since our value is the average of the corresponding areas.***

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method I: Classical Approach**
  - **We compare $z_0 = 2.00$ and $z_\alpha = 1.645$.**

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method I: Classical Approach**
  - **Since $z_0 > z_\alpha$, we can reject the null hypothesis.**

- In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?

- Method II: p-Value Approach

p-Value Approach for Hypothesis Testing for Proportion p

- Determine the p-value.

p-Value Approach for Hypothesis Testing for Proportion p

- Determine the p-value.
  - p-Value is the area below the standard normal curve
    - Left-tailed: $P(z < z_0)$
    - Right-tailed: $P(z > z_0)$
    - Two-tailed:

      $P(z < -|z_0| \text{ or } z > |z_0|)$

p-Value Approach for Hypothesis Testing for Proportion p

- Determine the p-value.
  - p-Value is the area
    - Left-tailed: $P(z < z_0)$
    - Right-tailed: $P(z > z_0)$
    - Two-tailed:

$$P(\, z < -|z_0|) + P(z > |z_0|)$$

p-Value Approach for Hypothesis Testing for Proportion p

- Determine the p-value.
  - p-Value is the area
    - Left-tailed: $P(z < z_0)$
    - Right-tailed: $P(z > z_0)$
    - Two-tailed: $2P(z < -|z_0|)$

p-Value Approach for Hypothesis Testing for Proportion p

- Determine the p-value.
  - p-Value is the area
    - Left-tailed: $P(z < z_0)$
    - Right-tailed: $P(z > z_0)$
    - Two-tailed: $2P(z < -|z_0|)$
- Compare p-value to α

p-Value Approach for Hypothesis Testing for Proportion p

- Determine the p-value.
  - p-Value is the area
    - Left-tailed: $P(z < z_0)$
    - Right-tailed: $P(z > z_0)$
    - Two-tailed: $2P(z < -|z_0|)$
- Reject the null hypothesis if p-value < α

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method II: p-Value Approach**
  - $P(z > 2.00) = P(z < -2.00)$
    $= 0.0228$

- In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?

- Method II: p-Value Approach
  - $P(z > 2.00) = 1 - P(z < 2.00)$

    $= 1 - 0.9772$

    $= 0.0228$

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method II: p-Value Approach**
  - **p-value = 0.0228**

- In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?

- Method II: p-Value Approach
  - Since the p-value, 0.0228, is less than the level of significance, 0.05, we can reject the null hypothesis.

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Method I: Classical Approach**
  - **Since $z_0 > z_\alpha$, we can reject the null hypothesis.**
- **Method II: p-Value Approach**
  - **Since the p-value, 0.0228, is less than the level of significance, 0.05, we can reject the null hypothesis.**

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Conclusion: At the 0.05 level of significance, we can reject the null hypothesis.**

- **In 2004, 72% of Americans said that sex education should be taught in middle school. In a June 2009 *Fox News* poll of 900 registered voters, 75% responded that sex education should be taught in middle school. Is there sufficient evidence to conclude that the percentage of Americans who say that sex education should be taught in middle school has increased?**

- **Conclusion: At the 0.05 level of significance, there is sufficient evidence to conclude that percentage of Americans who say that sex education should be taught in middle school has increased.**

We may choose to use either

- the Classical Approach

or

- the p-Value Approach.

Suggestion: Practice both

- the Classical Approach

and

- the p-Value Approach

as you work on the exercises. Since you must *always* obtain the same conclusion using either approach, using both methods as you study will help you to learn/understand each approach as well as confirm your conclusions.