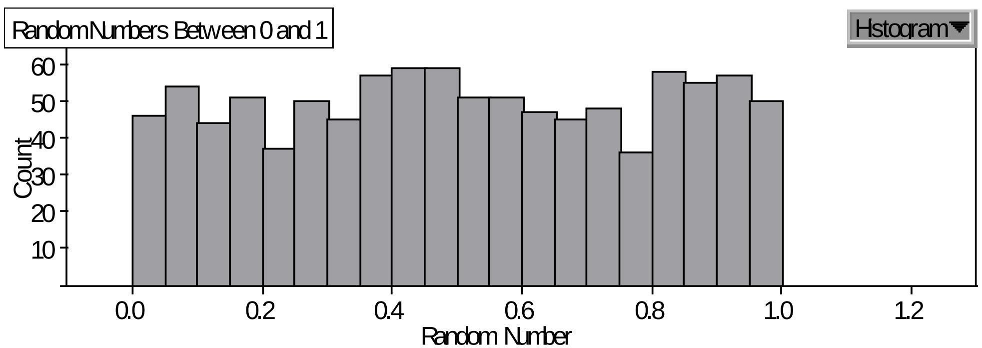
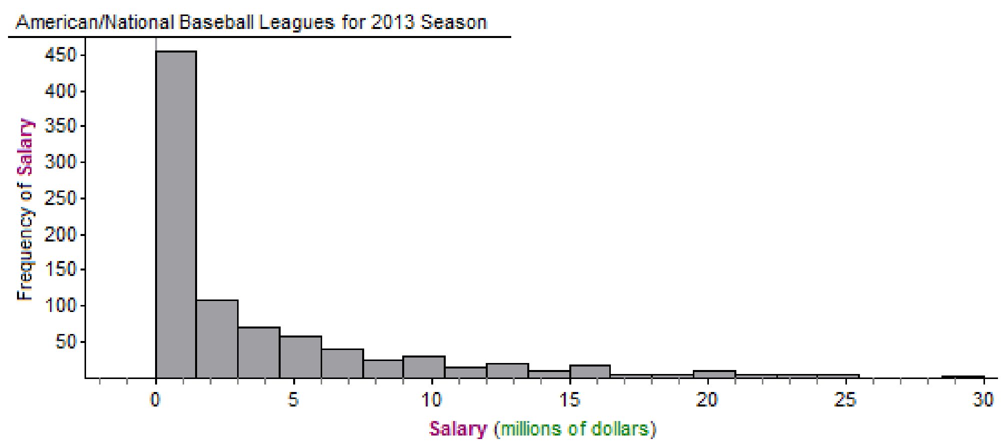
Normal Distribution

Normal Distribution

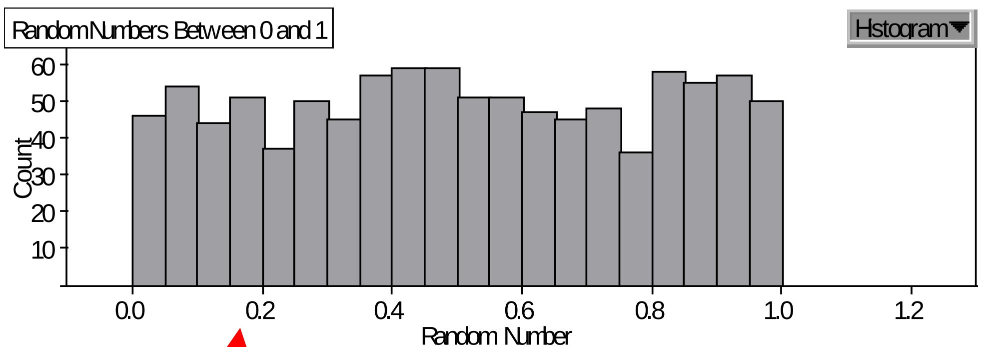
- What is the standard normal distribution?
- How can the standard normal distribution be used to answer questions about any distribution which is approximately normal?

What do you know about these distributions?

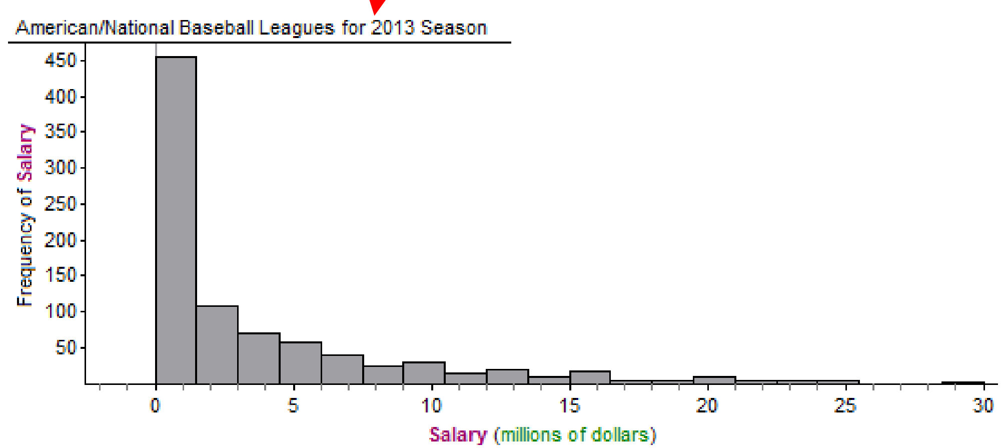




What do you know about these distributions?

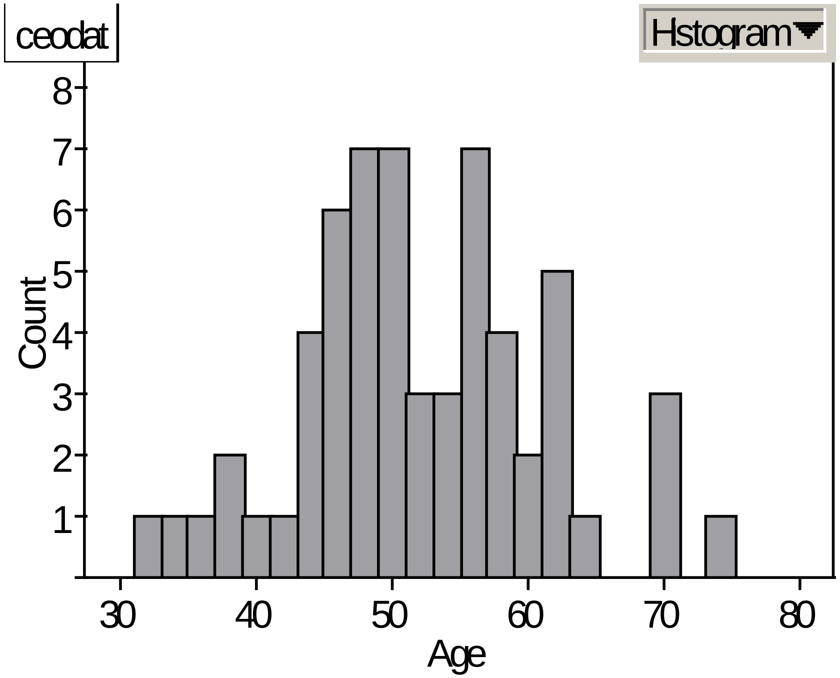


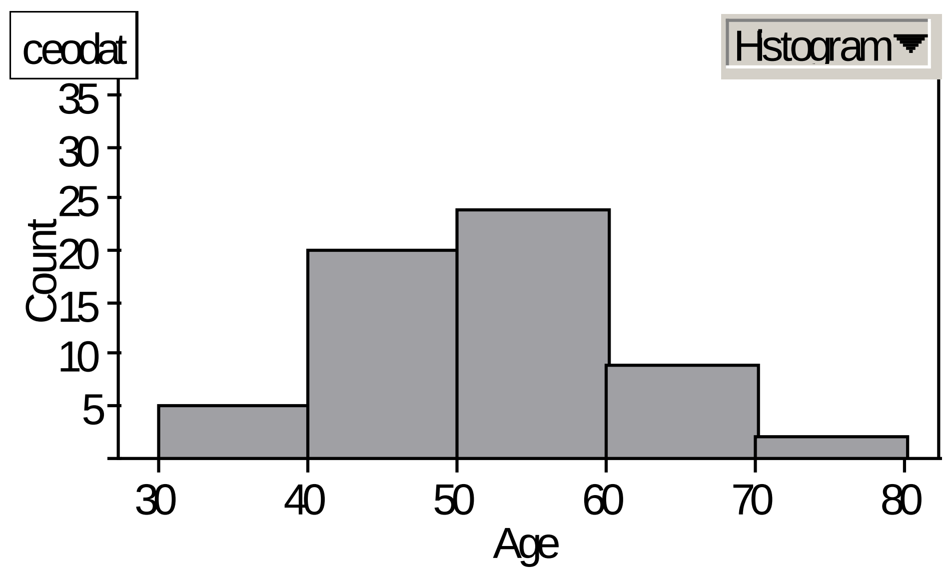
The distribution *below* is skewed to the right.



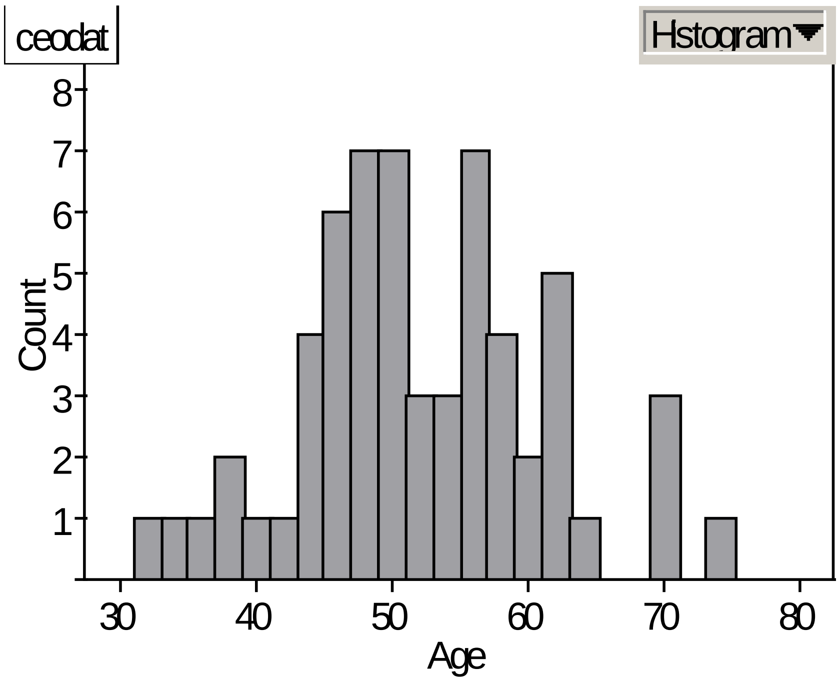
The distribution *above* is uniform.

What do you know about these distributions?

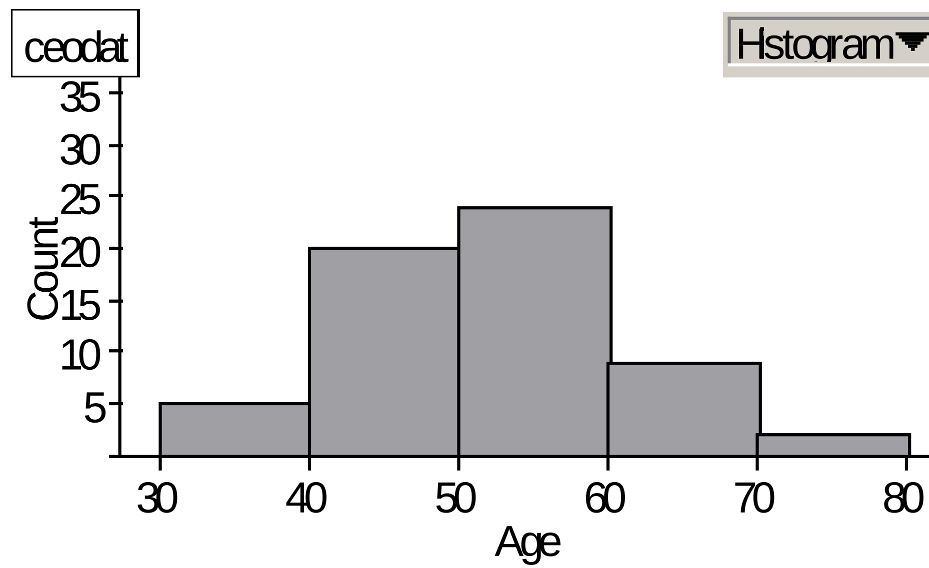




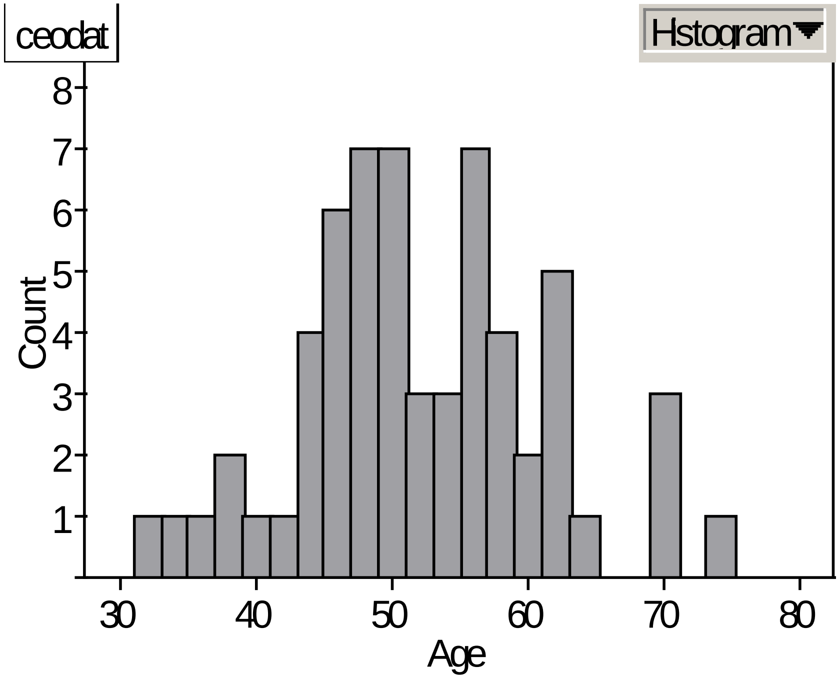
What do you know about these distributions?



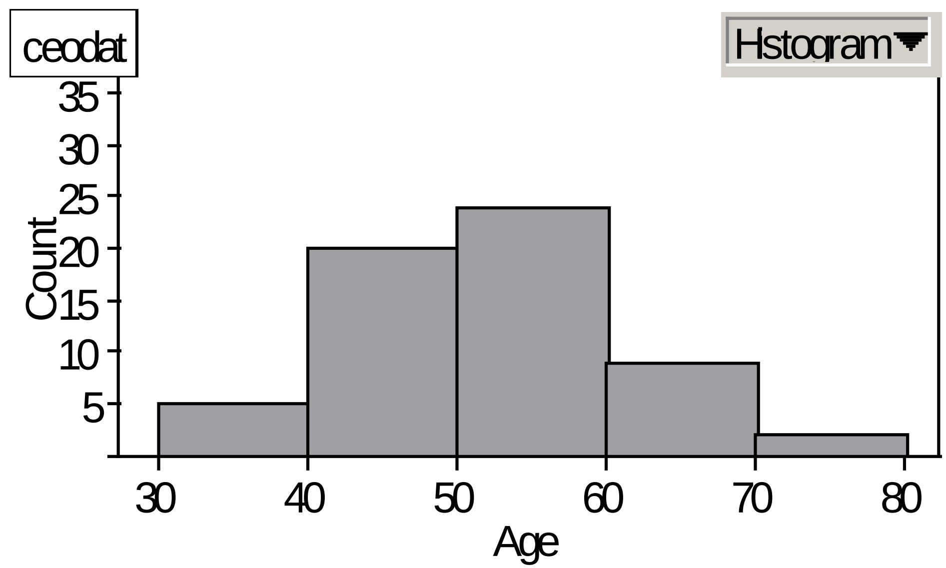
Each distribution is approximately normal.



What do you know about these distributions?

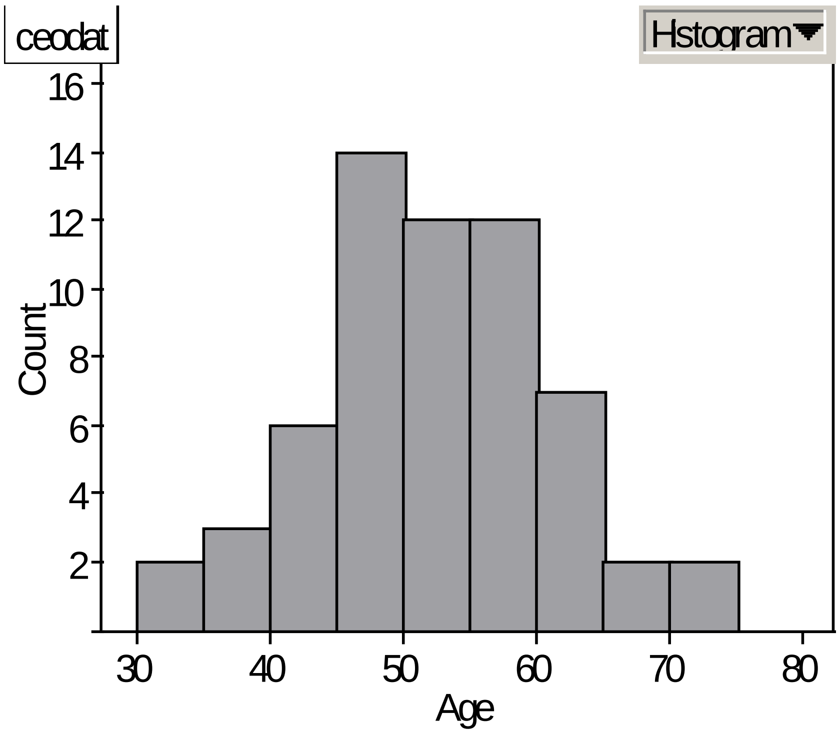


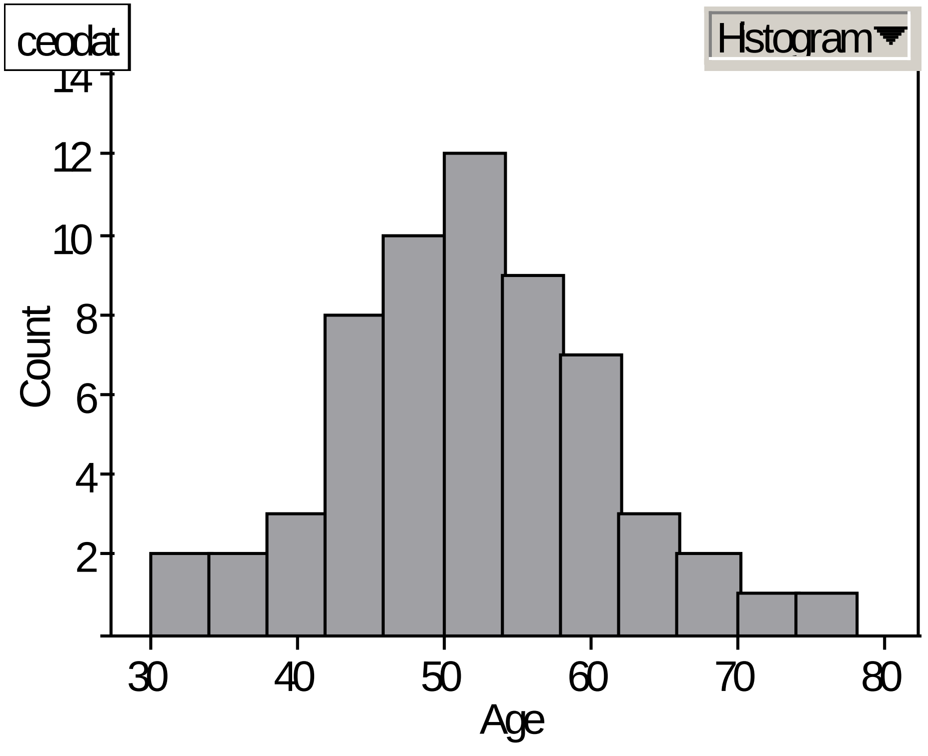
Each distribution is approximately normal.



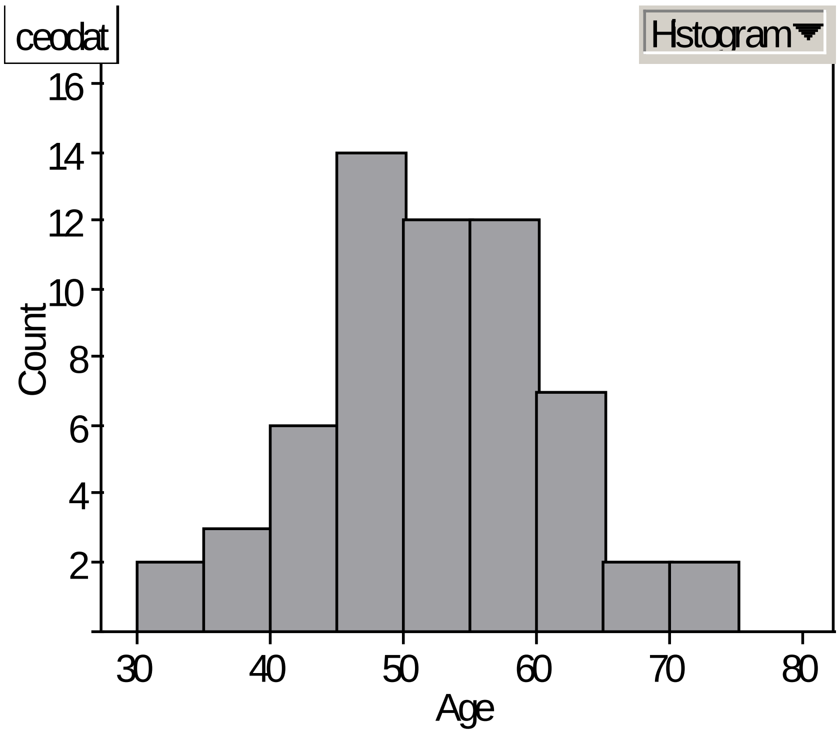
Neither distribution is perfectly normal.

What do you know about these distributions?

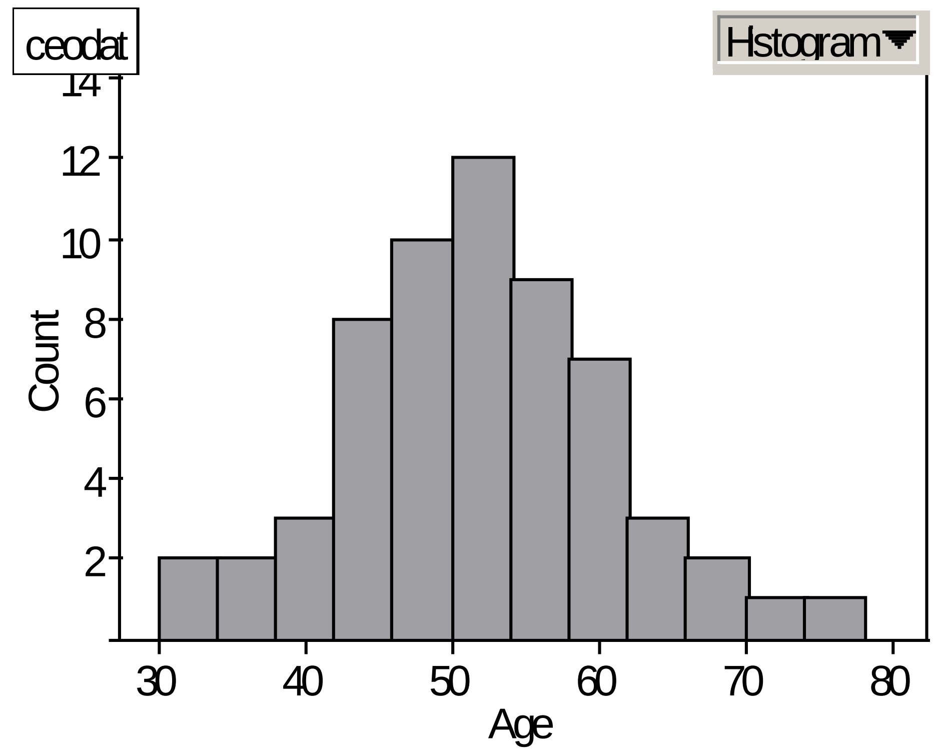




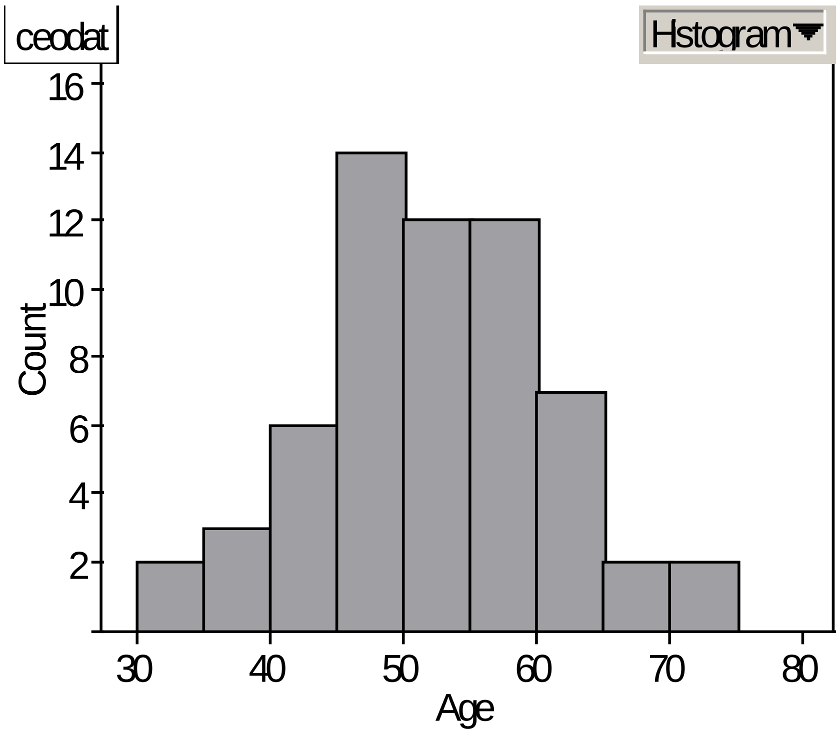
What do you know about these distributions?



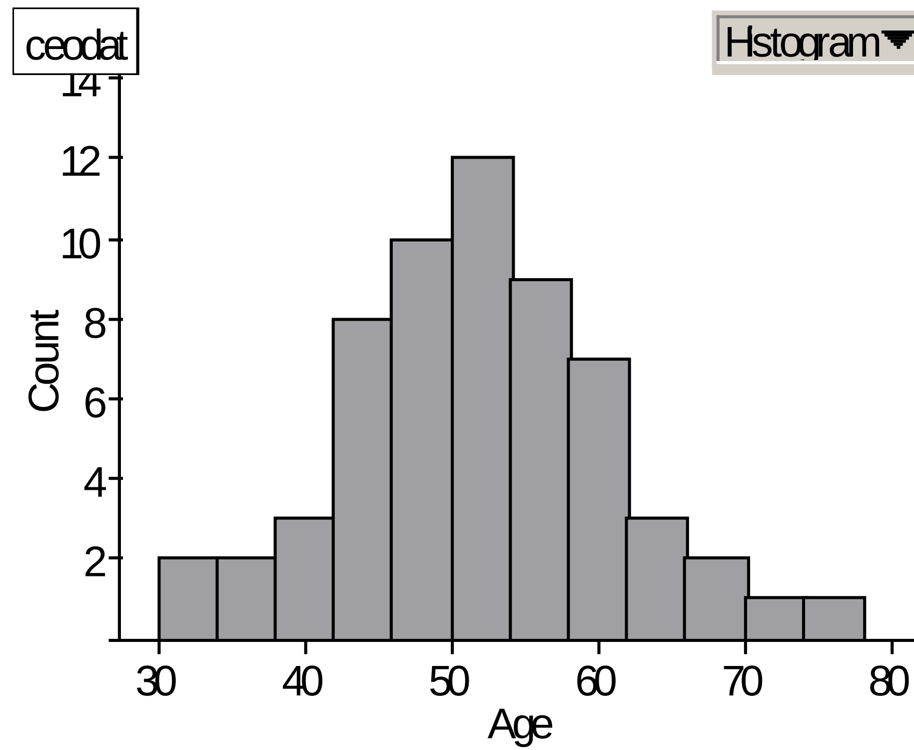
Each distribution is approximately normal.



What do you know about these distributions?



Each distribution is approximately normal.



Neither distribution is perfectly normal.

Can we answer the question posed below?

- Suppose the average speed for a group of animals is 35.5 miles per hour with a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?

Can we answer the question posed below?

- Suppose the average speed for a group of animals is 35.5 miles per hour with a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?
- No.

Why can't we answer the question posed below?

- Suppose the average speed for a group of animals is 35.5 miles per hour with a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?
- Although we know the mean and standard deviation, we do not have any data and we do not know the distribution type.

Can we answer the question posed below?

- The average height of a group of female students was 64.8 inches with a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?

Can we answer the question posed below?

- The average height of a group of female students was 64.8 inches with a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?
- **No.**

Why can't we answer the question posed below?

- The average height of a group of female students was 64.8 inches with a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?
- Although we know the mean and standard deviation, we do not have any data and we do not know the distribution type.

Can we answer the question posed below?

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. What percentage of bolts produced will have a diameter greater than 0.3 inch?

Can we answer the question posed below?

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. What percentage of bolts produced will have a diameter greater than 0.3 inch?
- No.

Why can't we answer the question posed below?

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. What percentage of bolts produced will have a diameter greater than 0.3 inch?
- Although we know the mean and standard deviation, we do not have any data and we do not know the distribution type.

Normal Distribution

Normal Distribution

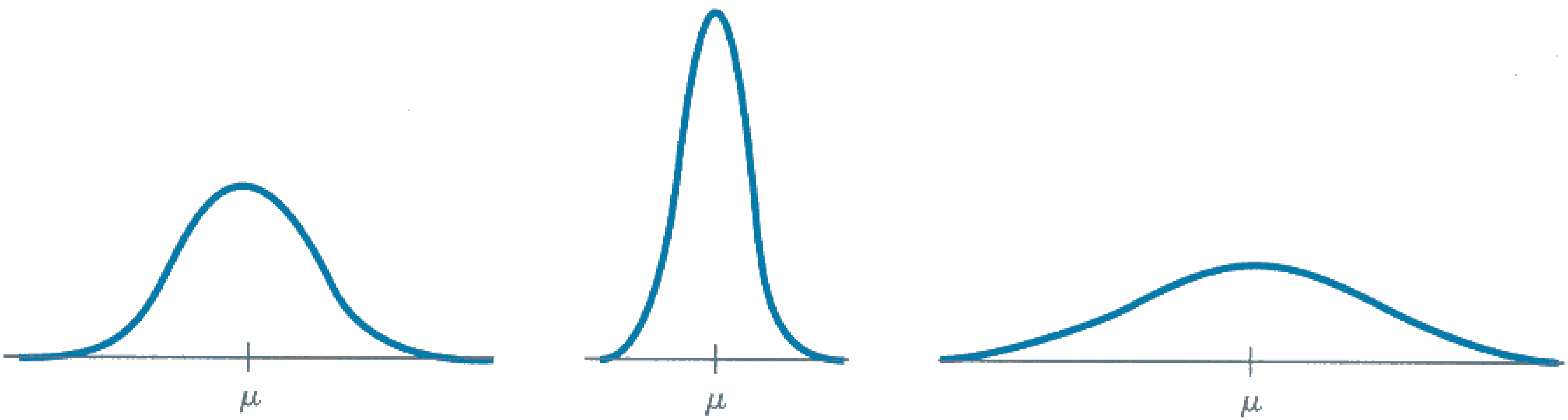
- All normal distributions have the same basic shape.

Normal Distribution

- All normal distributions have the same basic shape.

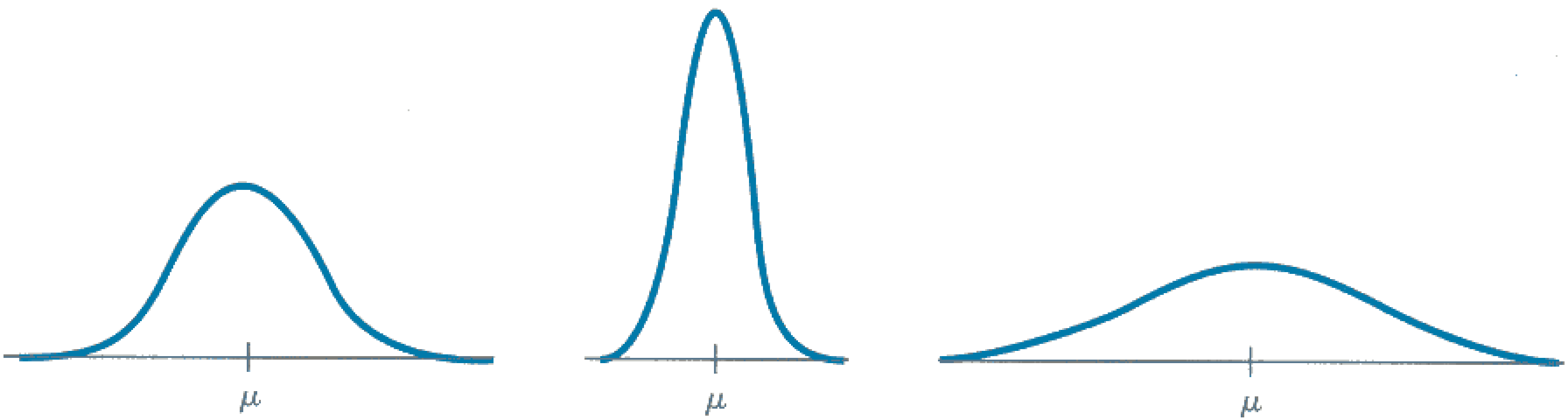
What is the shape of a Normal Distribution?

Normal Distribution



Three normal distributions

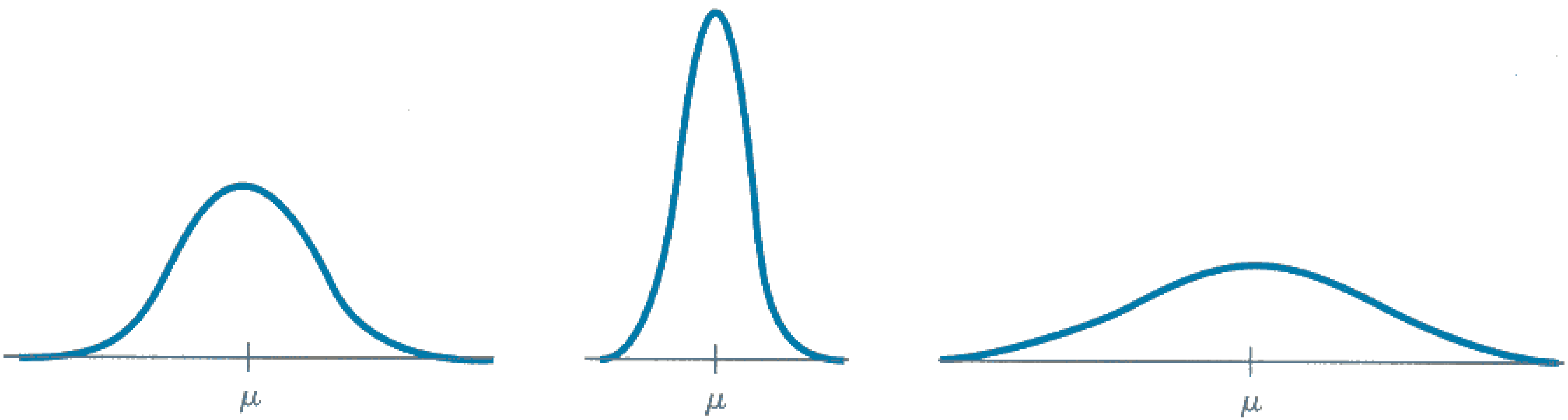
Normal Distribution



Three normal distributions

Each distribution given above is a normal distribution. They differ in shape since they each have a different standard deviation.

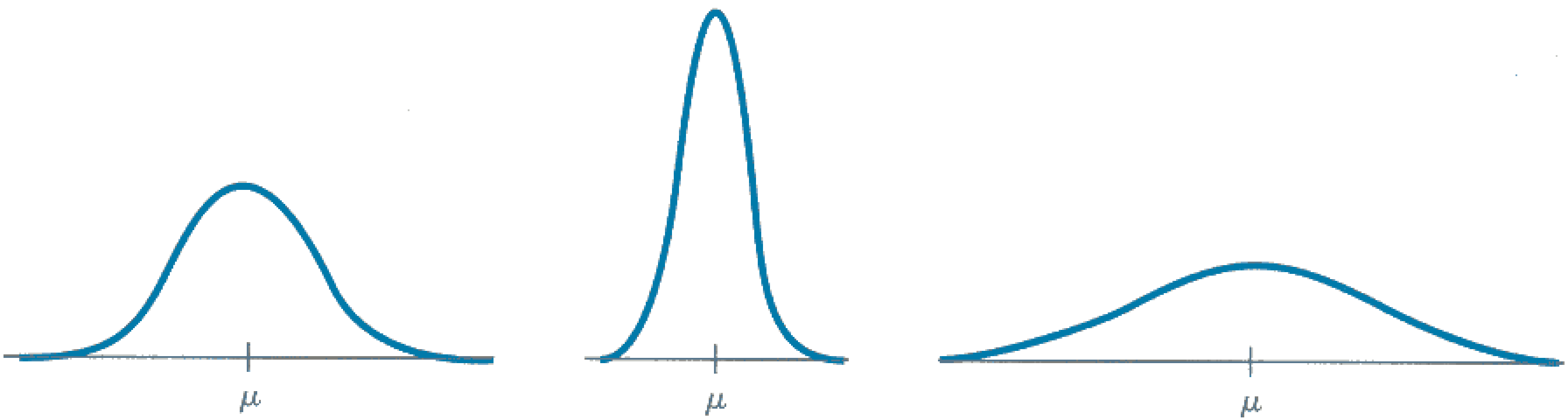
Normal Distribution



Three normal distributions

The distribution with the least spread has the smallest standard deviation.

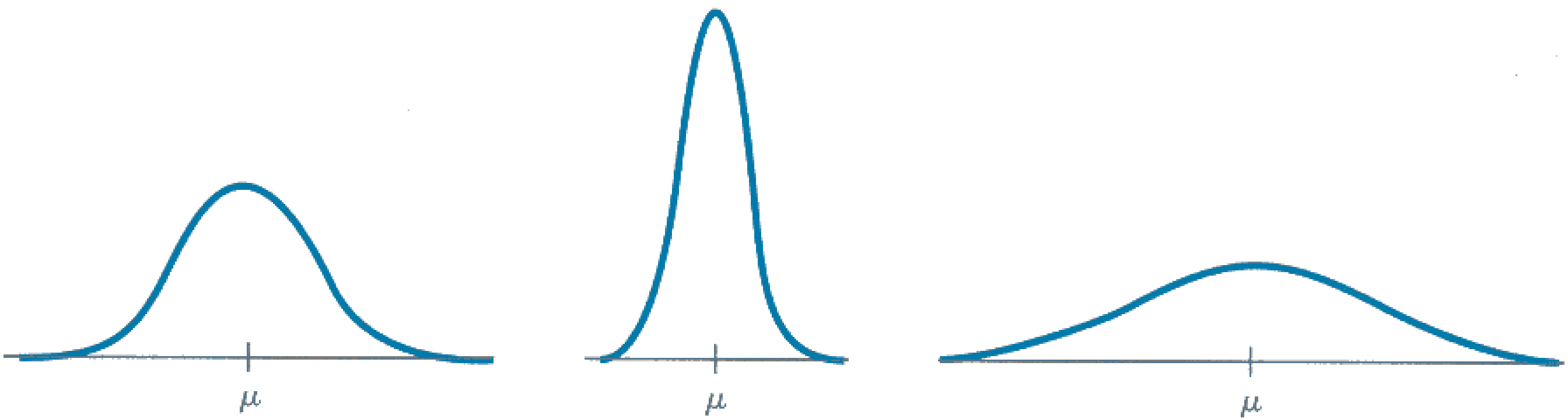
Normal Distribution



Three normal distributions

The distribution with the least spread (the one which appears to be more "compact") has the smallest standard deviation.

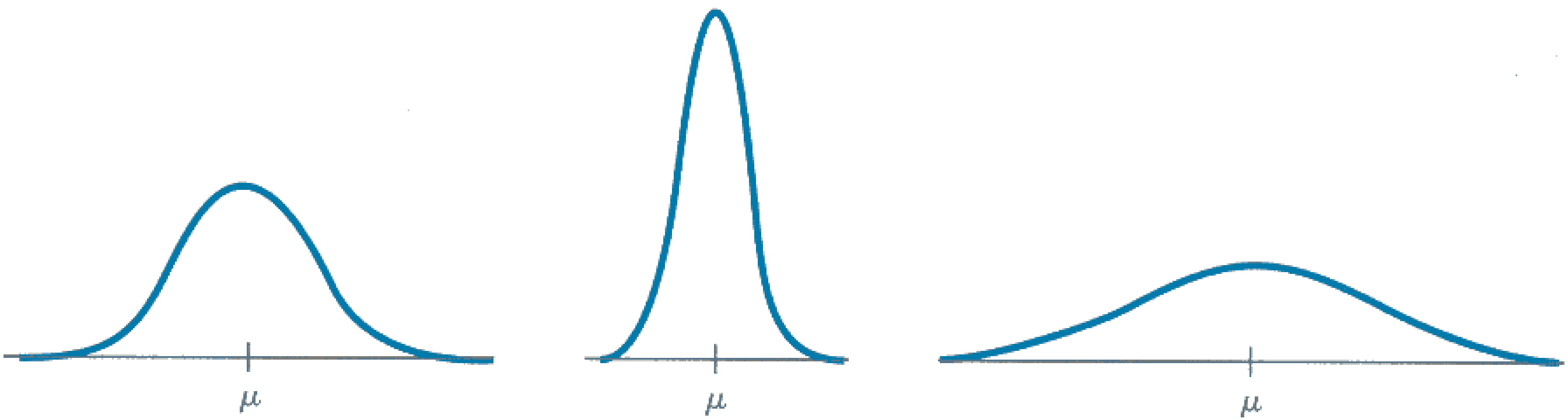
Normal Distribution



Three normal distributions

The distribution with the greatest spread has the largest standard deviation.

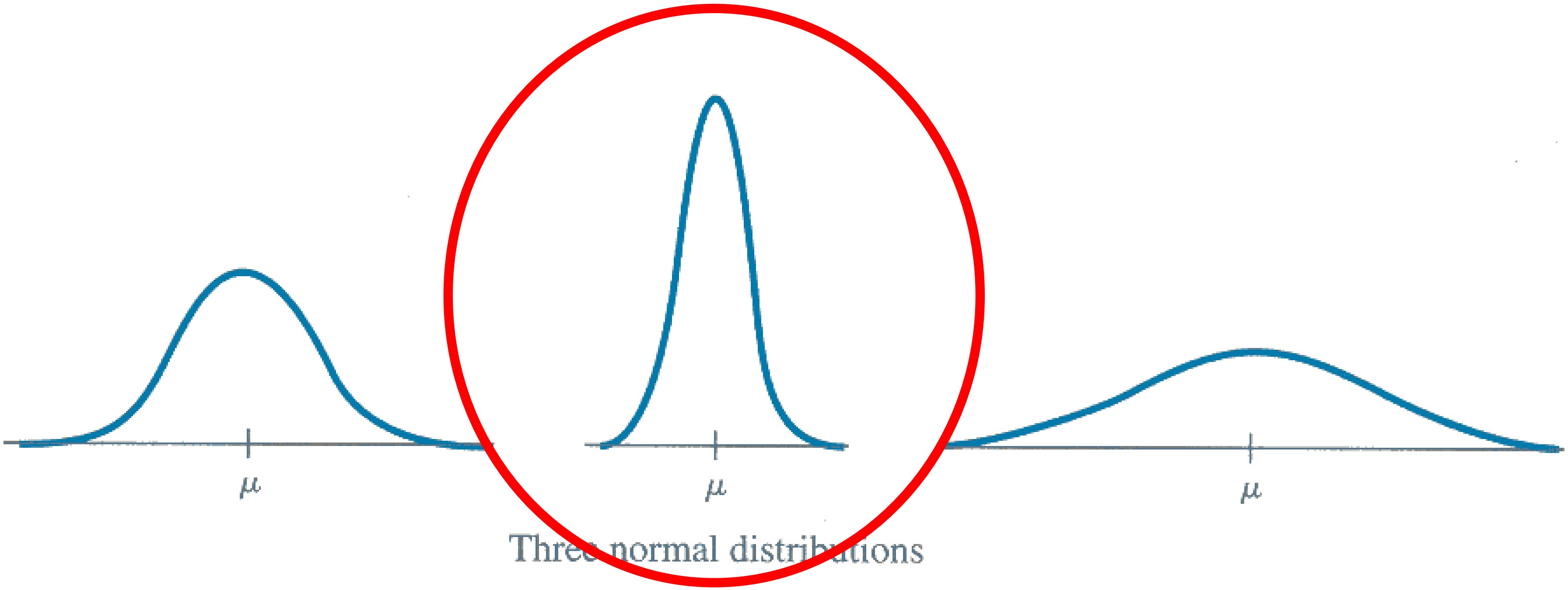
Normal Distribution



Three normal distributions

The distribution with the greatest spread (the distribution which appears to be the widest) has the largest standard deviation.

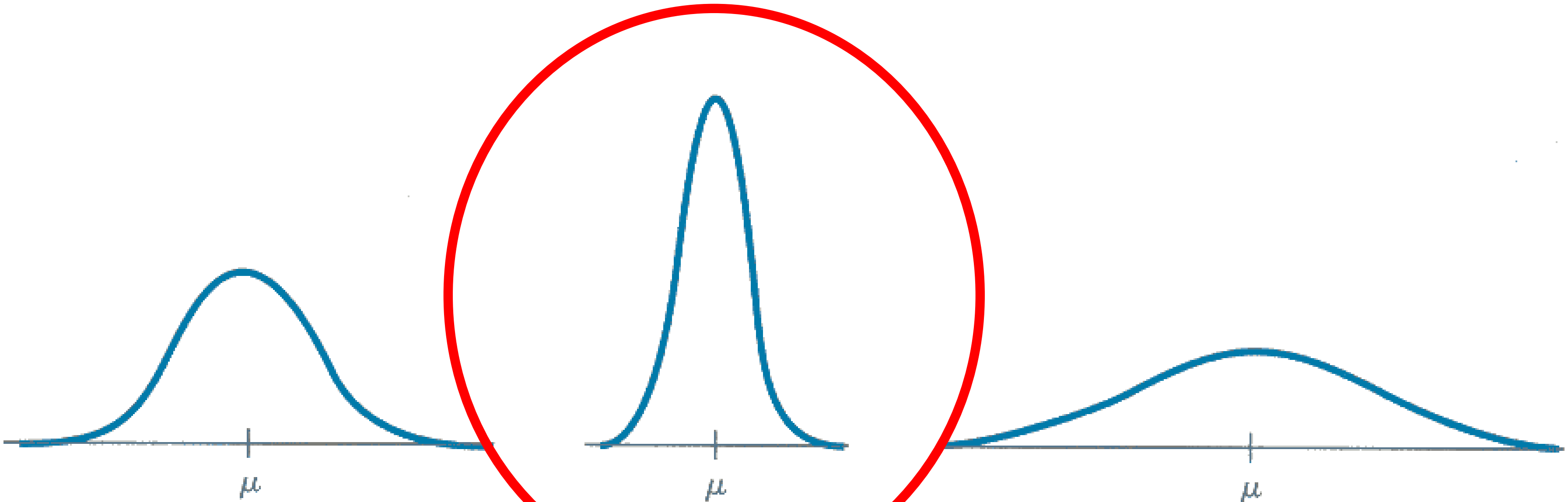
Normal Distribution



Three normal distributions

The distribution in the middle has the smallest standard deviation.

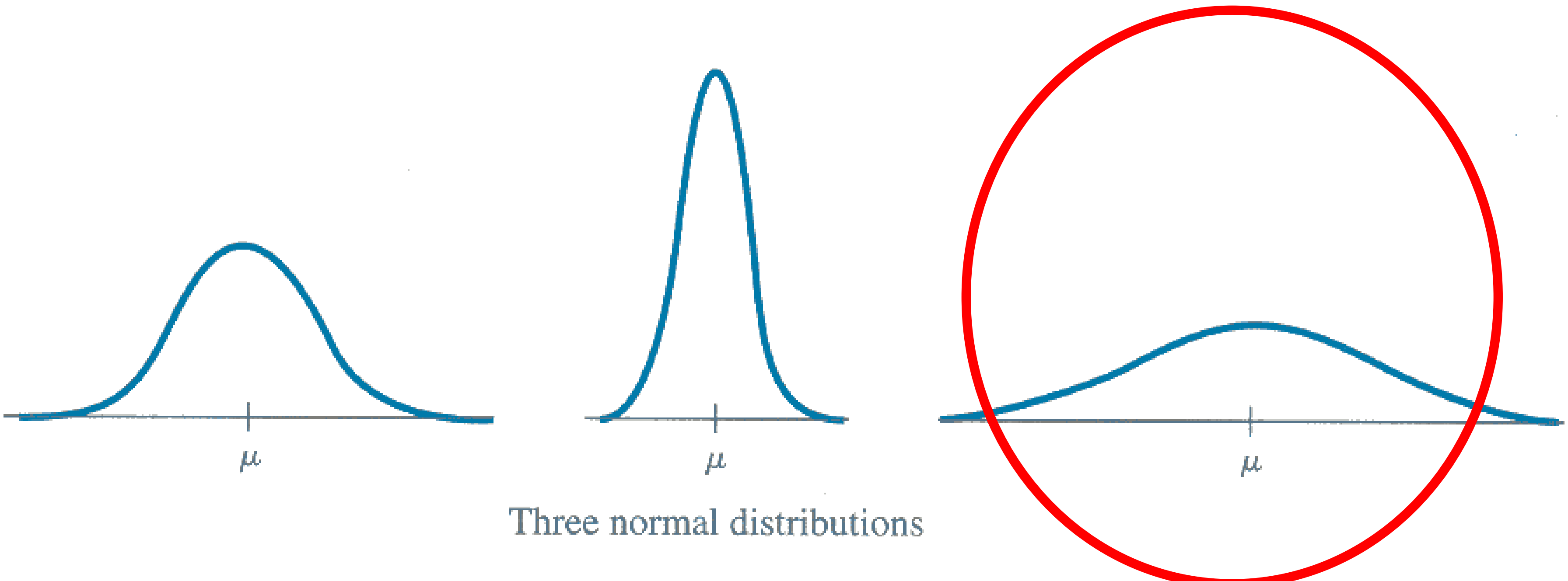
Normal Distribution



Three normal distributions

The distribution in the middle has the smallest standard deviation. This distribution has the least spread for the data values.

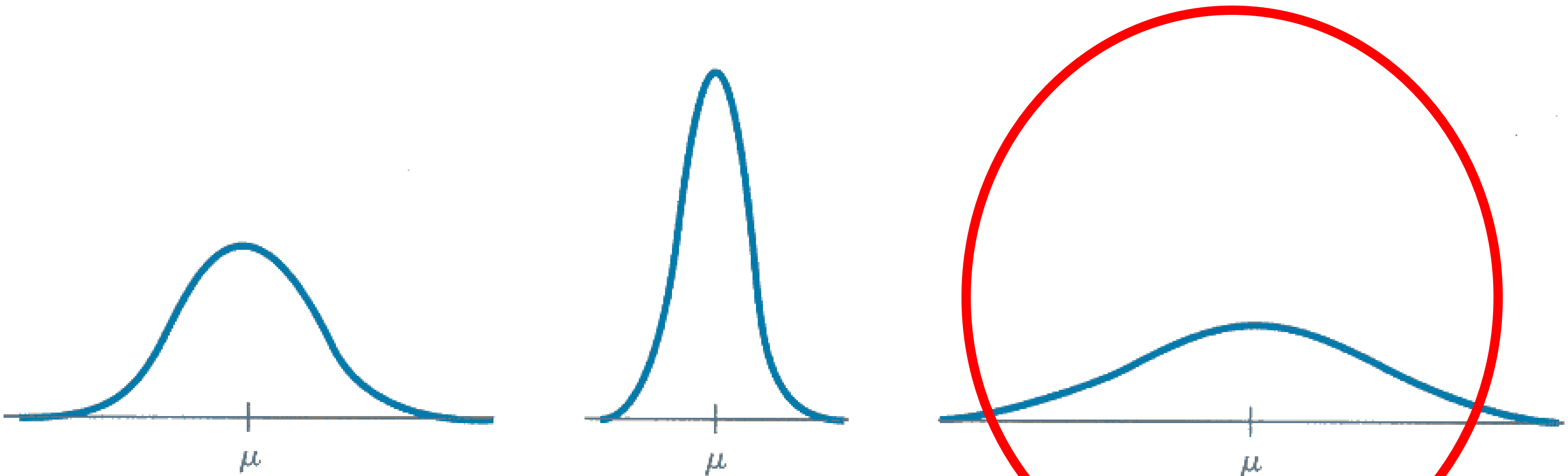
Normal Distribution



Three normal distributions

The distribution on the right has the largest standard deviation.

Normal Distribution



Three normal distributions

The distribution on the right has the largest standard deviation. This distribution has the greatest spread for the data values.

Normal Distribution

- All normal distributions have the same basic shape.

Normal Distribution

- All normal distributions have the same basic shape.

How would this help me?

Normal Distribution

- All normal distributions have the same basic shape.

How would this help me?

We can use z-scores to standardize values for any distribution.

Normal Distribution

- All normal distributions have the same basic shape.

How would this help me?

Z-scores center a distribution to a mean of 0 and rescale a distribution to have a standard deviation of 1.

Normal Distribution

- All normal distributions have the same basic shape.

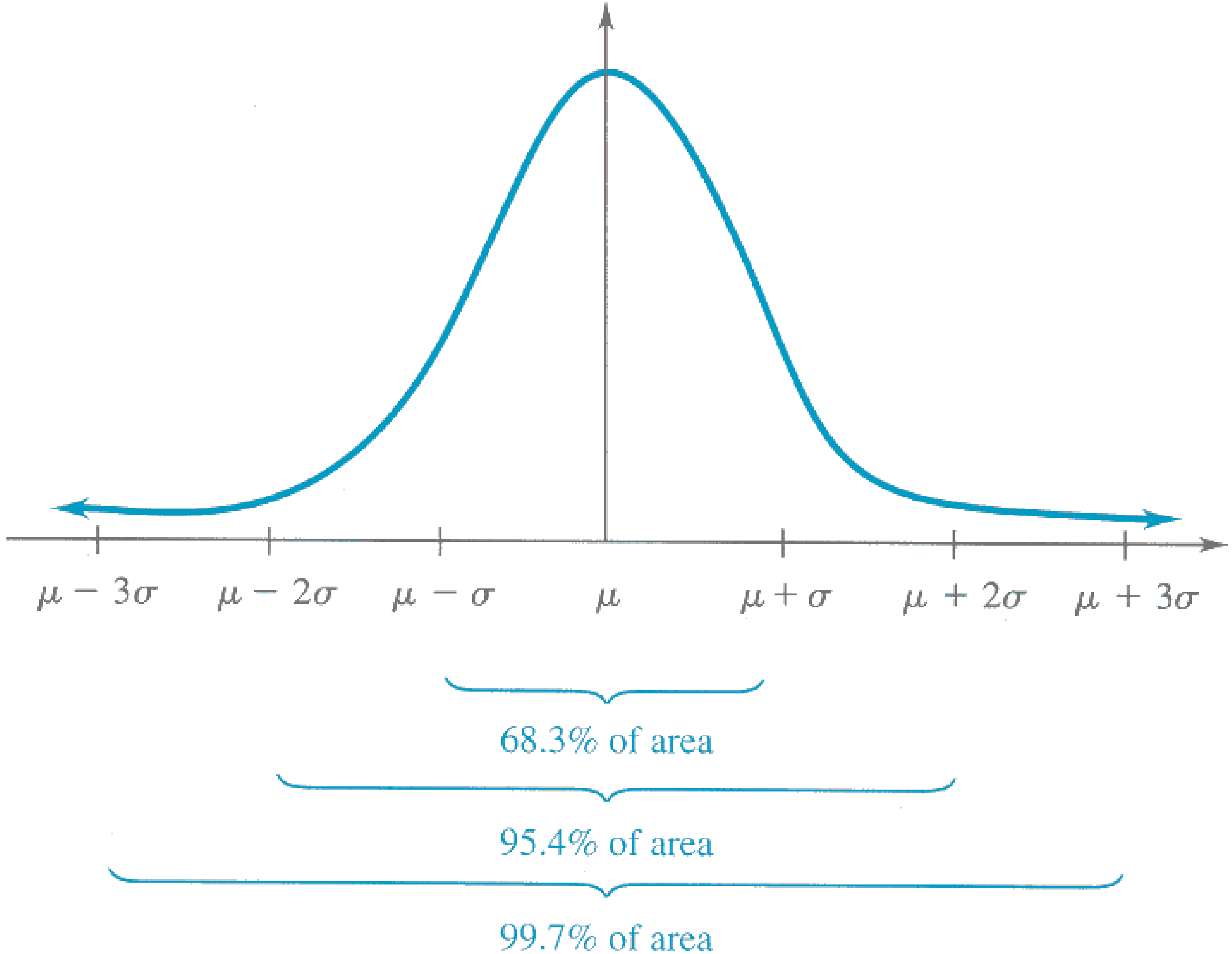
How would this help me?

Using z-scores, we can use the standard normal table to answer questions about *any* normal distribution.

Normal Distribution

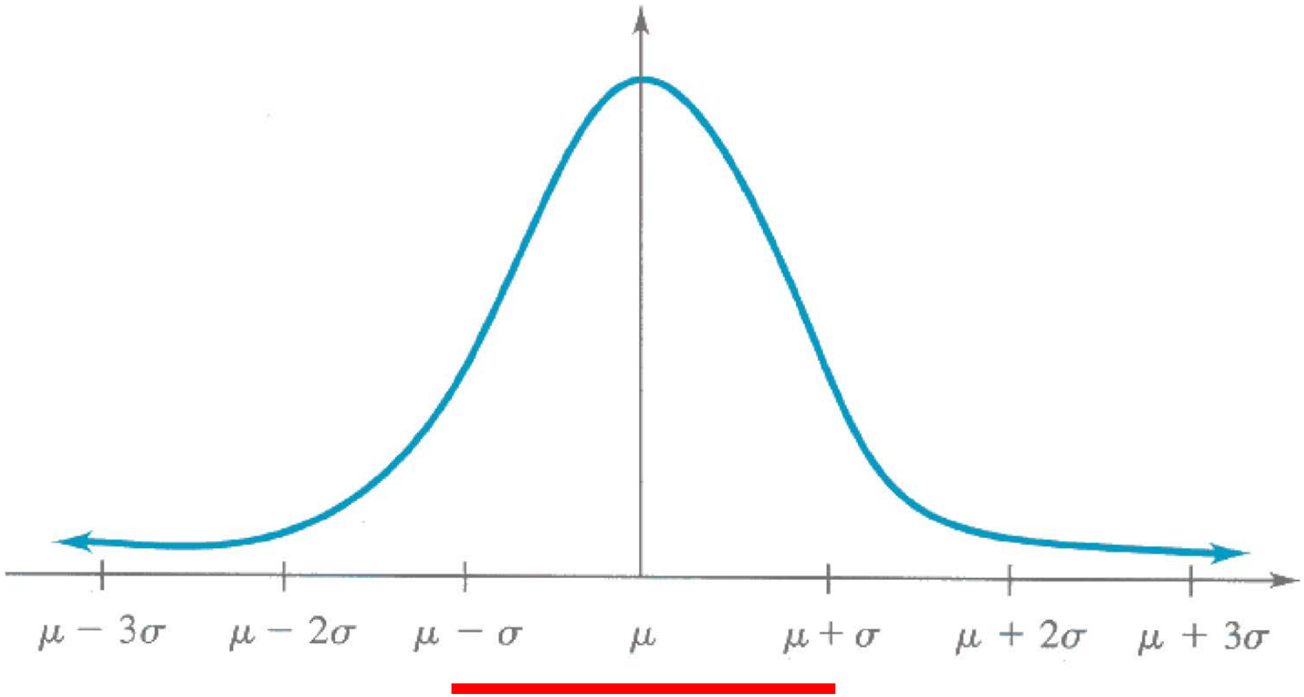
- All normal distributions have the same basic shape.

Normal Distribution*



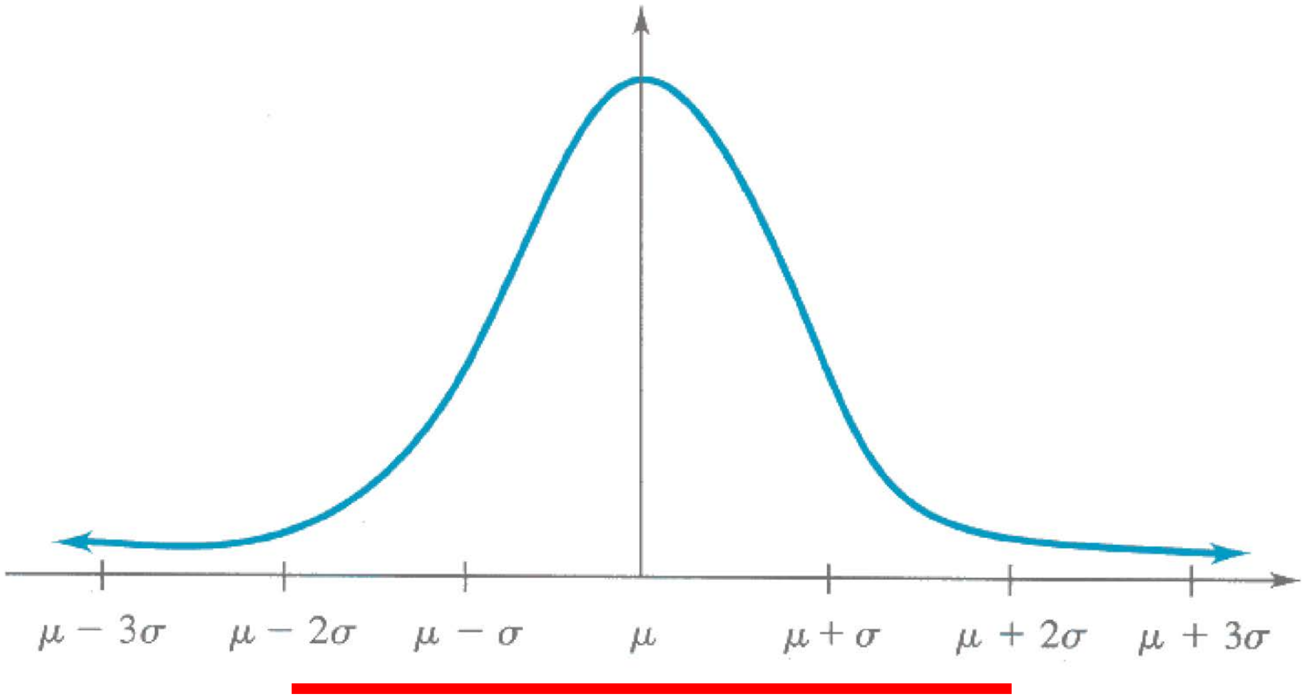
NOTE: The percentages given above have been rounded to one decimal place.

Normal Distribution



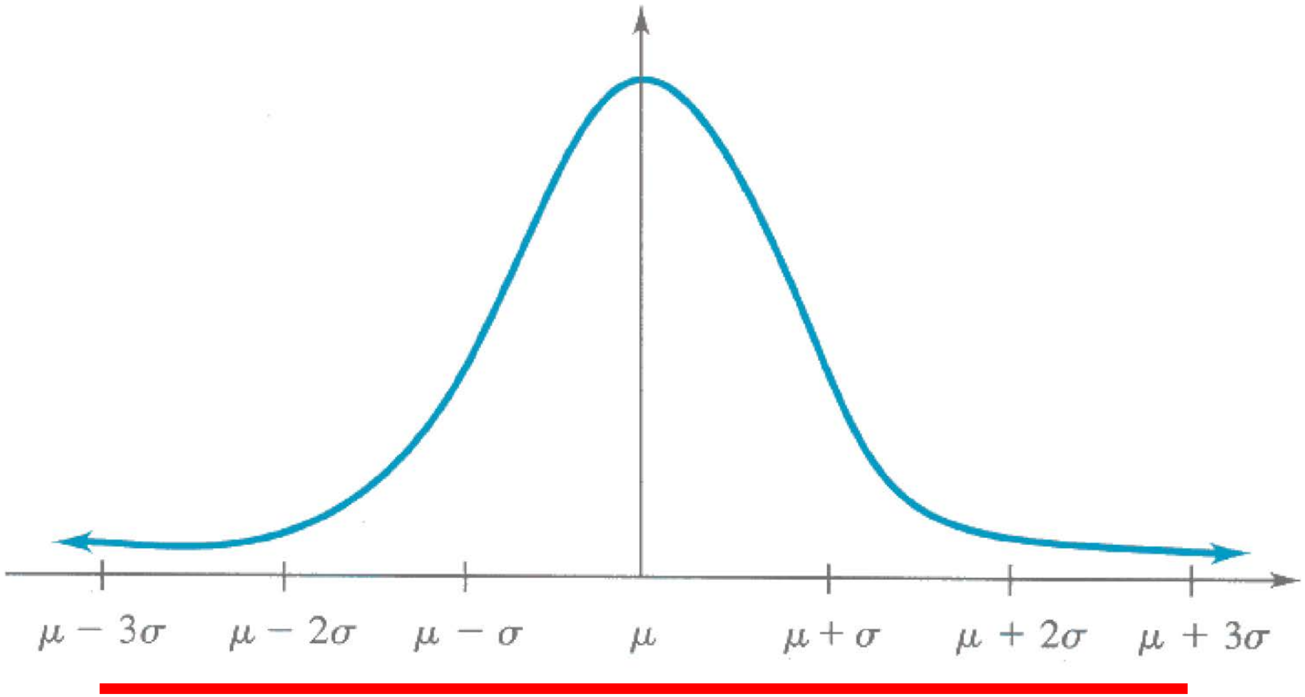
68.26% of the area below the standard normal curve lies within one standard deviation of the mean.

Normal Distribution



95.44% of the area below the standard normal curve lies within two standard deviations of the mean.

Normal Distribution



99.74% of the area below the standard normal curve lies within three standard deviations of the mean.

Normal Distribution

- Since all normal distributions have the same basic shape, we can use the Standard Normal Distribution in our analysis of any Normal Distribution.

Normal Distribution

- Since all normal distributions have the same basic shape, we can use the Standard Normal Distribution in our analysis of any Normal Distribution.

- The Standard Normal Distribution has a mean of 0 and a standard deviation of 1.

Standard Normal Distribution

- The mean is 0.
- The standard deviation is 1.
- The variable along the horizontal axis of the graph is called a z-score.
- The total area under the curve is equal to 1, that is 100%.

Standard Normal Distribution

- The mean is 0.
- The standard deviation is 1.
- The variable along the horizontal axis of the graph is called a z-score.
- The total area under the curve is equal to 1, that is 100%.

Now, we need to learn how to work with the Standard Normal Distribution.

Standard Normal Distribution

- Use to determine
  - Percentage of values to the left of a selected z-score
  - Probability that a randomly selected value of the random variable is below a selected z-score
  - z-score that corresponds to a particular percentage or probability

Standard Normal Distribution

- Use to determine
  - Percentage of values to the left of a selected z-score
  - Probability that a randomly selected value of the random variable is below a selected z-score
  - z-score that corresponds to a particular percentage or probability

How?

Standard Normal Distribution

- Use to determine
  - Percentage of values to the left of a selected z-score
  - Probability that a randomly selected value of the random variable is below a selected z-score
  - z-score that corresponds to a particular percentage or probability

Use the Standard Normal Table

Standard Normal Distribution

- Use to determine
  - Percentage of values to the left of a selected z-score
  - Probability that a randomly selected value of the random variable is below a selected z-score
  - z-score that corresponds to a particular percentage or probability

Use the symmetry of Normal curves

Standard Normal Distribution

- The values of the z-scores on the Standard Normal table have two (2) decimal places.
- The areas below the Standard Normal Curve to the left of specified z-scores are recorded to four (4) decimal places.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 1.03$

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- We look up $z = 1.03$ on the Standard Normal Distribution table (provided in the back of the textbook).

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- How do we do that?

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- Below the z column, we find the row marked with 1.0 and then, moving along that row, we find the value in the 0.03 column.

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- Below the z column, we find the row marked with 1.0 and then, moving along that row, we find the value in the 0.03 column. **Note: $1.0 + 0.03 = 1.03$.**

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- What does $1.0 + 0.03 = 1.03$ tell us about how to use the Standard Normal table?

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- The whole number and first decimal place, 1.0, are determined by the row of the table and the second decimal place, 0.03, is determined by selecting the appropriate column.

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- We find that the value 0.8485 lies in the position where the 1.0 row *meets* the 0.03 column.

Standard Normal Distribution

- Determine the percentage of values *below*
  - $z = 1.03$
- Using the Standard Normal table, we see that the amount of area to the left (below) of $z = 1.03$ is 0.8485 square units. So, 84.85% of values are below $z = 1.03$.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 2.67$

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 2.67$
- We determine the value on the table where the 2.6 row meets the 0.07 column.

Standard Normal Distribution

- Determine the percentage of values below
  - z = 2.67
- Looking up z = 2.67 on the Standard Normal table, we find that the amount of area below the standard normal curve to the left (below) of z = 2.67 is 0.9962 square units.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 2.67$
- 99.62% of values in the Standard Normal Distribution are below $z = 2.67$.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 0.08$

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 0.08$
- We determine the value on the table where the 0.0 row meets the 0.08 column.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 0.08$
- The amount of area below the Standard Normal curve to the left (below) of $z = 0.08$ is 0.5319 square units.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = 0.08$
- 53.19% of values in the Standard Normal Distribution are below $z = 0.08$.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -0.67$

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -0.67$
- We determine where the -0.6 row meets the 0.07 column.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -0.67$
- The value 0.2514 is the value in the position where the -0.6 row meets the 0.07 column.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -0.67$
- The amount of area below the Standard Normal Curve to the left of $z = -0.67$ is 0.2514 square units.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -0.67$
- 25.14% of values in the Standard Normal Distribution are below (less than) $z = -0.67$.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -2.34$

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -2.34$
- 0.0096 is the value on the Standard Normal table where the -2.3 row meets the 0.04 column.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -2.34$
- 0.0096 square units is the amount of area below the Standard Normal Curve to the left of $z = -2.34$.

Standard Normal Distribution

- Determine the percentage of values below
  - $z = -2.34$
- 0.96% of values in the Standard Normal Distribution are less than $z = -2.34$.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 94.06%

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 94.06%
- The area below the Standard Normal Curve is 0.9406 square units.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 94.06%
- We look up 0.9406 in the body of the table.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 94.06%
- When we find the value 0.9406 in the body of the table, we use the row label, 1.5, and the column heading, 0.06, to determine the z-score.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 94.06%
- When we find the value 0.9406 in the body of the table, we add the row label, 1.5, and the column heading, 0.06, to determine the z-score, $z = 1.56$.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 94.06%
- 94.06% of values in the Standard Normal Distribution are less than z = 1.56.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 85.08%

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 85.08%
- In order to determine the z-score corresponding to the area of 0.8508 square units, we look up 0.8508 in the body of the Standard Normal table.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 85.08%
- The value 0.8508 on the Standard Normal table is the value where the 1.0 row meets the 0.04 column.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 85.08%
- The value 0.8508 on the Standard Normal table corresponds to the z-score z = 1.04.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 85.08%
- 85.08% of values in the Standard Normal Distribution are less than z = 1.04.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 3.01%

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 3.01%
- In order to determine the z-score, we look for 0.0301 in the body of the Standard Normal table.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 3.01%
- The value 0.0301 in the body of the Standard Normal table is the value where the -1.8 row meets the 0.08 column.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 3.01%
- The value 0.0301 in the body of the Standard Normal table corresponds to the z-score -1.88.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 3.01%
- 3.01% of values in the Standard Normal are less than the z-score -1.88.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- Examining the Standard Normal table, we see that 0.2500 is not a value given in the body of the table.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- The closest values to 0.2500 in the Standard Normal table are 0.2514 and 0.2483.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- We note that 0.2514 has more than 25% of the area to the left of the corresponding z-score and 0.2483 has less than 25% of the area to the left of the corresponding z-score.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- We also note that these values are not equidistant from 0.2500. So, unlike the values provided in Example 6 on Page 291 of the text, we cannot just *split the difference*.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- What do we do?

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- We think about the question.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- We want to determine the z-score for which 25% of the values lie below. Since the area corresponding to $z = -0.67$ is 0.2514 square units, 25% of the area lies below this z-score.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 25%
- So, we take the $z = -0.67$ since 25% of the values in the Standard Normal Distribution lie below this value.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 48%

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 48%
- Since 0.4800 is not a value in the body of the Standard Normal table, we find the value in the table that is closest to 0.4800 but not less than 0.4800.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 48%
- The value in the Standard Normal table that is closest to 0.4800 but not less than 0.4800 is 0.4801.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 48%
- The z-score corresponding to 0.4801 is $z = -0.05$.

Standard Normal Distribution

- Determine the z-score that has the given percentage of values below it.
  - 48%
- 48% of the values in the Standard Normal Distribution are less than z = -0.05.

Standard Normal Distribution

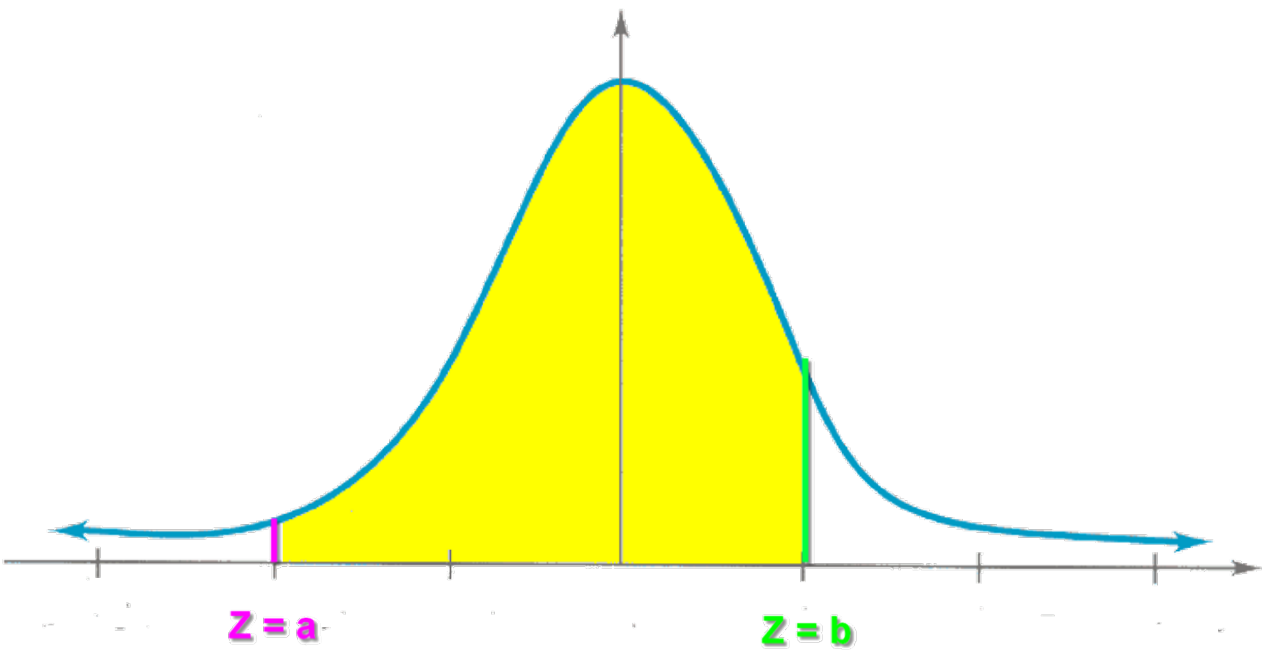
- Determine the percentage of values in the Standard Normal Distribution that fall between
  - 2.5 and 1

Standard Normal Distribution

- Determine the percentage of values in the Standard Normal Distribution that fall between
  - 2.5 and 1
- How do we determine the percentage of values between two z-scores?

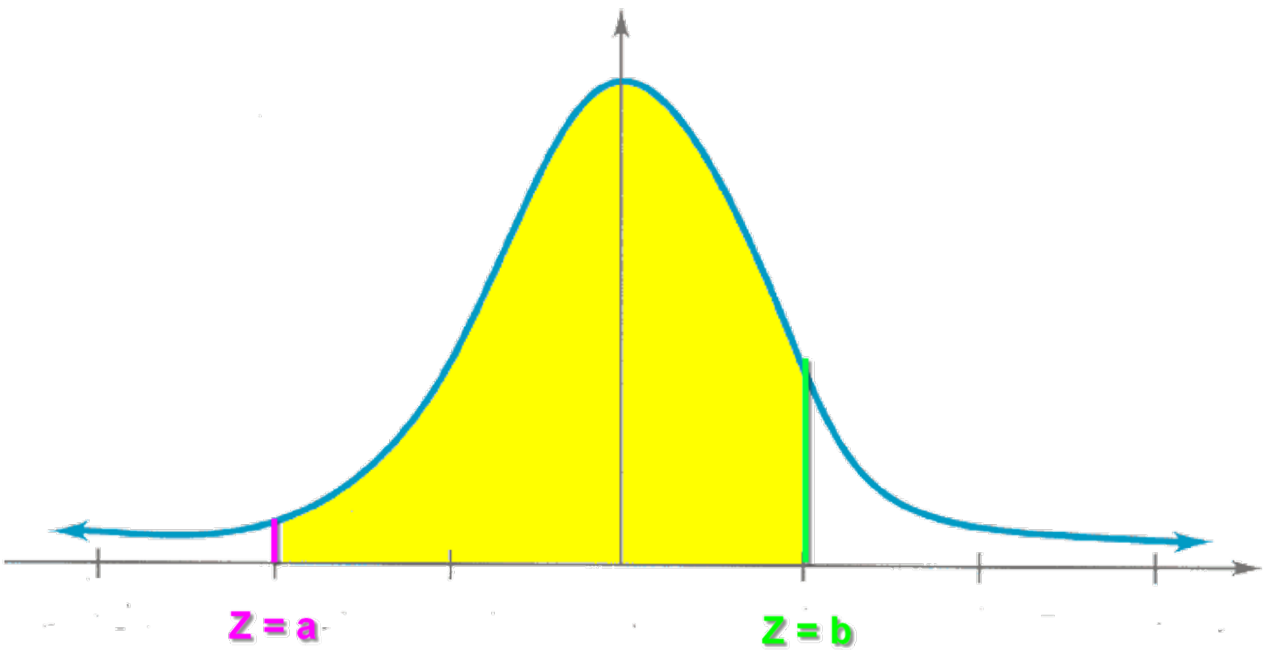
Standard Normal Distribution

- If we have two z-scores, for example z = a and z = b, for which we determine the corresponding areas below the Standard Normal Curve, subtracting the areas allows us to determine the amount of area *between* the two z-scores.



Standard Normal Distribution

- $P(a < z < b) = P(z < b) - P(z < a)$



Standard Normal Distribution

- Determine the percentage of values in the Standard Normal Distribution that fall between
  - 2.5 and 1
- We must determine the area corresponding to $z = 2.5$ and $z = 1$. We subtract the area corresponding to $z = 1$ from the area corresponding to $z = 2.5$.

Standard Normal Distribution

- Determine the percentage of values in the Standard Normal Distribution that fall between
  - 2.5 and 1
- $P(z < 2.5) = 0.9938$
- $P(z < 1) = 0.8413$
- $P(1 < z < 2.5) = P(z < 2.5) - P(z < 1)$

$$= 0.9938 - 0.8413$$
$$= 0.1525$$

Standard Normal Distribution

- Determine the percentage of values in the Standard Normal Distribution that fall between
  - 2.5 and 1
- 15.25% of values in the Standard Normal Distribution are between 2.5 and 1.

Standard Normal Distribution

- Determine the percentage of values in the Standard Normal Distribution that fall between
  - -2 and 2

Standard Normal Distribution

- Determine the percentage of values in the Standard Normal Distribution that fall between
  - -2 and 2
- $P(z < 2) = 0.9772$
- $P(z < -2) = 0.0228$
- $P(-2 < z < 2) = P(z < 2) - P(z < -2)$

  $= 0.9772 - 0.0228$

  $= 0.9544$

Standard Normal Distribution

- Determine the percentage of values in the Standard Normal Distribution that fall between
  - -2 and 2
- 95.44% of values in the Standard Normal Distribution are between -2 and 2.

Standard Normal Distribution

- Determine the percentage of values in the standard normal distribution that fall between
  - 0 and 1.58

Standard Normal Distribution

- Determine the percentage of values in the standard normal distribution that fall between
  - 0 and 1.58
- $P(z < 1.58) = 0.9429$
- $P(z < 0) = 0.5000$
- $P(0 < z < 1.58) = P(z < 1.58) - P(z < 0)$

  $= 0.9429 - 0.5000$

  $= 0.4429$

Standard Normal Distribution

- Determine the percentage of values in the standard normal distribution that fall between
  - 0 and 1.58
- 44.29% of values in the Standard Normal Distribution are between 0 and 1.58.

Standard Normal Distribution

- Determine the percentage of values in the standard normal distribution that fall between
  - -1.23 and –0.15

Standard Normal Distribution

- Determine the percentage of values in the standard normal distribution that fall between
  - -1.23 and –0.15
- $P(z < -0.15) = 0.4404$
- $P(z < -1.23) = 0.1093$
- $P(-1.23 < z < -0.15) = P(z < -0.15) - P(z < -1.23)$

$$= 0.4404 - 0.1093$$
$$= 0.3311$$

Standard Normal Distribution

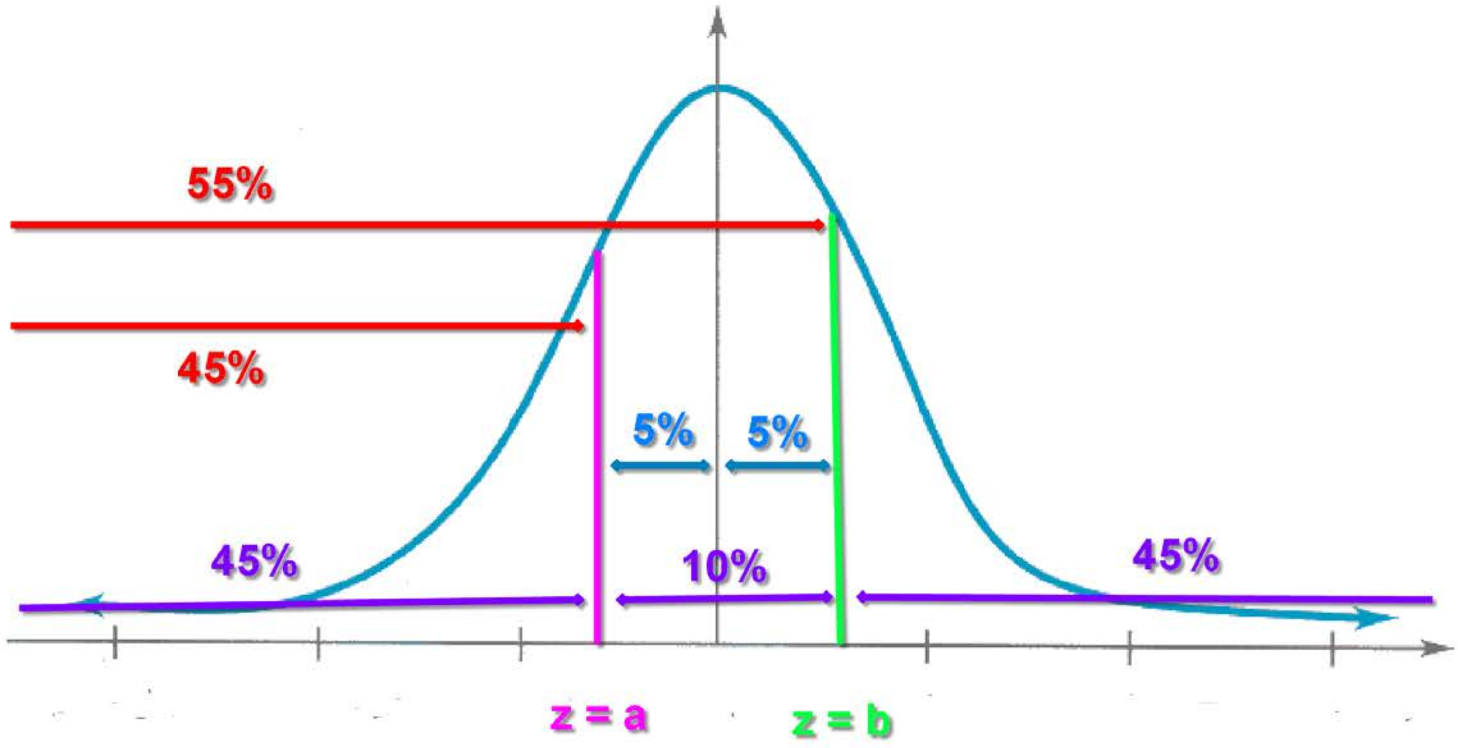
- Determine the percentage of values in the standard normal distribution that fall between
  - -1.23 and –0.15
- 33.11% of values in the Standard Normal Distribution are between -1.23 and -0.15.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores



Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- If we want 10% of the z-scores to be in the middle then 5% of the values must be to the left of the mean and 5% of the values must be to the right of the mean.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- If we want to have 5% of the values between the desired z-score and the mean then 45% of the values in the distribution must be to the left of this z-score.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- Using the symmetry of the Normal Distribution, we know that there is a corresponding z-score on the right-hand side of the mean as well.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- For the z-score to the right of the mean, 45% of the values are to the right of this z-score and 55% of values are to the left of this z-score.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- To determine the desired interval, we determine the z-scores that are as close as possible to 0.4500 and 0.5500 in the body of the Standard Normal table.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- $P(z < -0.12) = 0.4522$
- $P(z < -0.13) = 0.4483$
- $P(z < 0.13) = 0.5517$
- $P(z < 0.12) = 0.5478$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- $P(z < -0.12) = 0.4522$
- $P(z < 0.13) = 0.5517$
- $P(-0.12 < z < 0.13) = 0.0995$
  - We cannot use this pair of z-scores since less than 10% of values in the distribution are between them.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- $P(z < -0.13) = 0.4483$
- $P(z < 0.12) = 0.5478$
- $P(-0.13 < z < 0.12) = 0.0995$
  - We cannot use this pair of z-scores since less than 10% of values in the distribution are between them.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- $P(z < -0.12) = 0.4522$
- $P(z < 0.12) = 0.5478$
- $P(-0.12 < z < 0.12) = 0.0956$
  - We cannot use this pair of z-scores since less than 10% of values in the distribution are between them.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- $P(z < -0.13) = 0.4483$
- $P(z < 0.13) = 0.5517$
- $P(0.13 < z < 0.13) = 0.1034$
  - We can use this pair of z-scores since 10% of values in the distribution are between them.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- For the Standard Normal Distribution, the middle 10% of z-scores are between z = -0.13 and z = 0.13.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- For the Standard Normal Distribution, the middle 10% of z-scores are between $z = -0.13$ and $z = 0.13$.
- CAUTION: The z-scores are not always the same in magnitude.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 10% of the z-scores
- For the Standard Normal Distribution, the middle 10% of z-scores are between $z = -0.13$ and $z = 0.13$.
- The interval is $-0.13 < z < 0.13$.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 90% of the z-scores

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 90% of the z-scores
- To have 90% of the values in the middle, we need one z-score for which 95% of the values are to the left and one z-score for which 5% of the values are to the left.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 90% of the z-scores
- $P(z < -1.64) = 0.0505$
- $P(z < -1.65) = 0.0495$
- $P(z < 1.64) = 0.9495$
- $P(z < 1.65) = 0.9505$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 90% of the z-scores
- $P(z < -1.64) = 0.0505$
- $P(z < -1.65) = 0.0495$
- $P(z < 1.64) = 0.9495$
- $P(z < 1.65) = 0.9505$
- With these values, there will be two possible intervals.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 90% of the z-scores
- $P(z < -1.64) = 0.0505$
- $P(z < 1.65) = 0.9505$
- $P(-1.64 < z < 1.65) = 0.9000$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 90% of the z-scores
- $P(z < -1.65) = 0.0495$
- $P(z < 1.64) = 0.9495$
- $P(-1.65 < z < 1.64) = 0.9000$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 90% of the z-scores
- The intervals $-1.65 < z < 1.65$ and $-1.64 < z < 1.65$ contain the middle 90% of values in the Standard Normal Distribution.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 50% of the z-scores

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 50% of the z-scores
- In order to have 50% of values in the middle, we need one z-score for which 75% of values are to the left and one z-score for which 25% of values are to the left.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 50% of the z-scores
- $P(z < -0.68) = 0.2483$
- $P(z < -0.67) = 0.2514$
- $P(z < 0.67) = 0.7486$
- $P(z < 0.68) = 0.7517$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 50% of the z-scores
- $P(z < -0.68) = 0.2483$
- $P(z < -0.67) = 0.2514$
- $P(z < 0.67) = 0.7486$
- $P(z < 0.68) = 0.7517$
- With these values, there will be two possible intervals.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 50% of the z-scores
- $P(z < -0.68) = 0.2483$
- $P(z < 0.67) = 0.7486$
- $P(-0.68 < z < 0.67) = 0.5003$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 50% of the z-scores
- $P(z < -0.67) = 0.2514$
- $P(z < 0.68) = 0.7517$
- $P(-0.67 < z < 0.68) = 0.5003$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 50% of the z-scores
- For the Standard Normal Distribution, the intervals $-0.68 < z < 0.67$ and $-0.67 < z < 0.68$ contain the middle 50% of z-scores.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 40% of the z-scores

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 40% of the z-scores
- In order to have 40% of values in the middle, we need one z-score for which 70% of values are to the left and one z-score for which 30% of values are to the left.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 40% of the z-scores
- $P(z < -0.53) = 0.2981$
- $P(z < -0.52) = 0.3015$
- $P(z < 0.52) = 0.06985$
- $P(z < 0.53) = 0.7019$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 40% of the z-scores
- $P(z < -0.53) = 0.2981$
- $P(z < -0.52) = 0.3015$
- $P(z < 0.52) = 0.06985$
- $P(z < 0.53) = 0.7019$
- With these values, there will be two possible intervals.

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 40% of the z-scores
- $P(z < -0.53) = 0.2981$
- $P(z < 0.52) = 0.06985$
- $P(-0.53 < z < 0.52) = 0.4004$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 40% of the z-scores
- $P(z < -0.52) = 0.3015$
- $P(z < 0.53) = 0.7019$
- $P(-0.52 < z < 0.53) = 0.4004$

Standard Normal Distribution

- For a Standard Normal Distribution, determine the interval that contains
  - the middle 40% of the z-scores
- For the Standard Normal Distribution, the intervals that contain 40% of z-scores are $-0.53 < z < 0.52$ and $-0.52 < z < 0.53$.

Using Standard Normal Distribution

- Converting to *standard units* or *standardizing*
  - Two step process of recentering and rescaling
  - Turns any normal distribution into a standard normal distribution

Using Standard Normal Distribution

- Recenter the values of the normal distribution so that the mean is at 0

Using Standard Normal Distribution

- Recenter the values of the normal distribution so that the mean is at 0

How?

Using Standard Normal Distribution

- Recenter the values of the normal distribution so that the mean is at 0
  - Subtract the mean
    - So, given an x-value in the normal distribution of interest, we recenter the value by subtracting the mean μ,

$$x - \mu$$

Using Standard Normal Distribution

- Recenter the values of the normal distribution so that the mean is at 0
  - $x - \mu$ tells us how far the value x is from the mean μ

Using Standard Normal Distribution

- Rescale the distribution so that the standard deviation is 1
  - Divide $x - \mu$ by the standard deviation σ
    - Dividing $x - \mu$ by σ tells us how many σ's there are in the distance from the mean $x - \mu$

Standardizing a Normal Distribution

- Changing an x-value in the normal distribution into a z-score for the standard normal distribution
  - How far from mean: $x - \mu$
  - How many standard deviations: $\dfrac{x-\mu}{\sigma}$
- So, we take the z-score to be

$$z = \frac{x-\mu}{\sigma}$$

What is the difference between the following two exercises?

- **Suppose the average speed for a group of animals is 35.5 miles per hour with a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?**

- Suppose the average speed for a group of animals is 35.5 miles per hour with a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?
- Note: We do not know the distribution type and we have no data.

- **Suppose the speed for a group of animals is approximately normal with an average speed of 35.5 miles per hour and a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?**

- Suppose the speed for a group of animals is approximately normal with an average speed of 35.5 miles per hour and a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?
- Note: We know that this distribution is approximately normal.

- Suppose the speed for a group of animals is approximately normal with an average speed of 35.5 miles per hour and a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?
- Note: We can answer this question by recentering and rescaling the distribution to the Standard Normal Distribution.

- **Determine the solution for the exercise that we *can* solve based on the given information.**

- Suppose the speed for a group of animals is approximately normal with an average speed of 35.5 miles per hour and a standard deviation of 14.1 miles per hour. What speed separates the fastest 50% of the animals from the rest?
- We do not need to do any work in order to answer this question since the mean separates the fastest 50% of animals from the rest. So, the speed of 35.5 miles per hour separates the fastest 50% of animals from the rest.

What is the difference between the following two exercises?

- The average height of a group of female students was 64.8 inches with a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?

- **The average height of a group of female students was 64.8 inches with a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?**
- **<u>Note</u>: We do not know the distribution type and we have no data.**

- The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 $\frac{1}{2}$ feet and 6 feet tall?

- The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?
- Note: We know that this distribution is approximately normal.

- The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 $\frac{1}{2}$ feet and 6 feet tall?
- Note: We can answer this question by recentering and rescaling the distribution to the Standard Normal Distribution.

- **Determine the solution for the exercise that we *can* solve based on the given information.**

- **The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?**
- **Note: We must make sure that the units match before we use the z-score formula.**

- **The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?**
- **5 ½ feet is equivalent to 66 inches**
- **6 feet is equivalent to 72 inches.**

- **The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?**
- **$z_1 = (66 - 64.8)/2.5 = 0.48$**
- **$z_2 = (72 - 64.8)/2.5 = 2.88$**

- The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?
- $P(0.48 < z < 2.88) = 0.9980$

$$- 0.6844$$

$$= 0.3136$$

- **The height of a group of female students is approximately normal with an average of 64.8 inches and a standard deviation of 2.5 inches. What is the percentage of the students who are between 5 ½ feet and 6 feet tall?**
- **31.36% of students are between 5 ½ feet and 6 feet tall.**

What is the difference between the following two exercises?

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. What percentage of bolts produced will have a diameter greater than 0.3 inch?

- **A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. What percentage of bolts produced will have a diameter greater than 0.3 inch?**
- **<u>Note</u>: We do not know the distribution type and we have no data.**

- **A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter greater than 0.3 inch?**

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter greater than 0.3 inch?
- Note: We know that this distribution is approximately normal.

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter greater than 0.3 inch?
- Note: We can answer this question by recentering and rescaling the distribution to the Standard Normal Distribution.

- **Determine the solution for the exercise that we *can* solve based on the given information.**

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter greater than 0.3 inch?
- Note: We have "greater than" rather than "less than". "Greater than" corresponds to "the right" of the z-score.

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter **greater than** 0.3 inch?
- $P(z > a) = 1 - P(z < a)$

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter greater than 0.3 inch?
- We determine the z-score.
  - $z = (0.3 - 0.25)/0.02 = 2.5$

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter greater than 0.3 inch?
- $P(z > 2.5) = 1 - P(z < 2.5)$

$$= 1 - 0.9938$$

$$= 0.0062$$

- A machine produces bolts with an average diameter of 0.25 inch and a standard deviation of 0.02 inch. If the diameter of the bolts is approximately normal, what percentage of bolts produced will have a diameter greater than 0.3 inch?
- 0.62% of bolts will have a diameter greater than 0.3 inch.

- **For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?**

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- We need to determine the z-score, z_1, for which $P(z < z_1) = 0.0250$.
- We need to determine the z-score, z_2, for which $P(z < z_2) = 0.9750$.

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- For $P(z < z_1) = 0.0250$, $z_1 = -1.96$
- For $P(z < z_2) = 0.9750$, $z_2 = 1.96$

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- Now that we have the z-scores, z_1 = -1.96 and, z_2 = 1.96, we need to determine the x-scores; the x-score is the value in the original distribution.

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- For $z_1 = -1.96$,

$$-1.96 = (x_1 - 1100)/180.$$

- For $z_2 = 1.96$,

$$1.96 = (x_2 = 1100)/180.$$

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- To determine x_1, we solve the equation $-1.96 = (x_1 - 1100)/180$.
- To determine x_2, we solve the equation $1.96 = (x_2 = 1100)/180$.

NOTE: Since $z = (x - \mu)/\sigma$, we can multiply both sides of the equation by σ,

$$z\sigma = x - \mu$$

and add μ to both sides of the equation,

$$z\sigma + \mu = x.$$

Therefore, we can use $x = \mu + \sigma z$ when we want to determine the x-score.

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- To determine x_1, we solve the equation

$$x_1 = 1100 + (-1.96)(180).$$

- To determine x_2, we solve the equation

$$x_2 = 1100 + (1.96)(180).$$

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- Solving the equations, we obtain

$$x_1 = 747.2 \quad \text{and} \quad x_2 = 1452.8.$$

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- Since SAT scores are given as natural numbers, we round 747.2 to 747 and we round 1452.8 to 1452.

- For students entering the University of Florida in a recent year, the distribution of SAT scores was roughly normal, with mean 1100 and standard deviation 180. The middle 95% of the SAT scores were between what two values?
- The middle 95% of SAT scores are between 747 points and 1452 points.