Probability of Compound Events

Probability of Compound Events

- What is the complement of an event?
- What are disjoint events?
- How do we determine the probability of the union of two events or the intersection of two events?

Complement of an Event

- Given an event E, the complement of event E can be denoted as
  - E^C
  - E´
  - $\bar{E}$
- NOTE: In this PowerPoint, we will use E´ to denote the complement of the event E.

Complement of an Event

- Given an event E, the complement of event E, E´, is the event that consists of all simple events in the sample space that are not simple events in event E.

Complement of an Event

- That is, given an event E, the complement of event E, E´, is the event containing all simple events that are not contained in event E.

Complement of an Event

- You may find it helpful to think of event E´ as the set of simple events *not in E* or *outside of E*.

Complement of an Event

- We can use a Venn Diagram to picture this.

What is a Venn Diagram?

What is a Venn Diagram?

- A Venn Diagram is a picture representation that is used to represent sets.
- For probability,
  - We use a rectangle to represent the sample space, S.
  - We use circles to represent events such as E.

Venn Diagram for the Sample Space

- The sample space S.

What is a Venn Diagram?

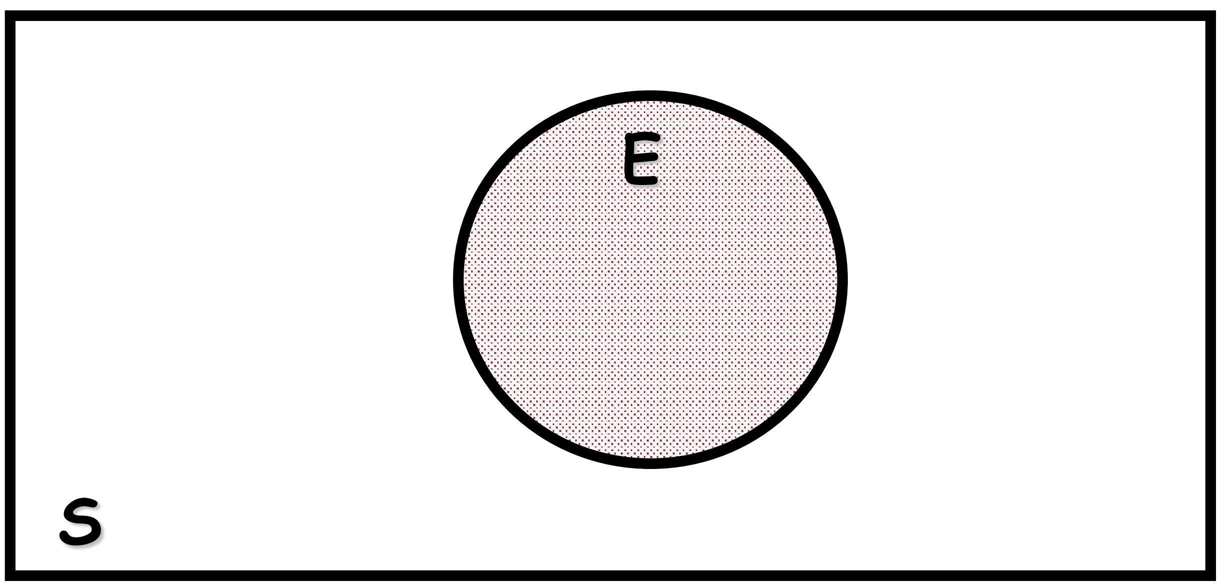
- We use a circle and the region inside the circle placed within the rectangle to represent an event.

What is a Venn Diagram?

- To represent the event E in the sample space S,
  - We create a circle inside the rectangle to define the space for the event E
  - We shade the inside to the circle to represent the event E
  - We label the circle as E

Venn Diagram for an Event

- The event E.

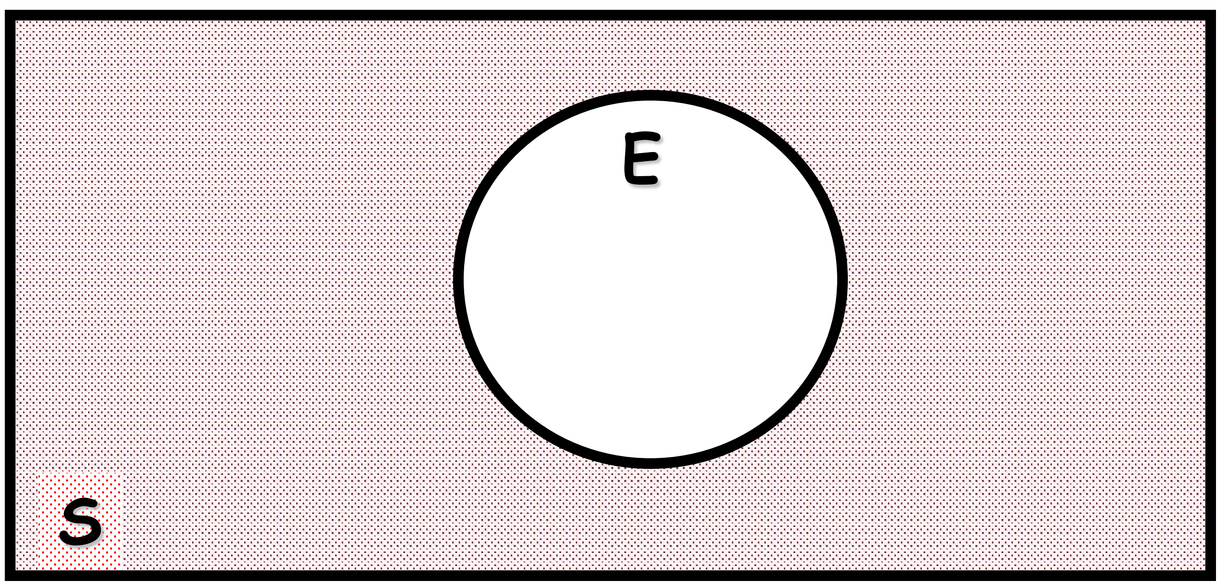


Complement of an Event

- To create the Venn Diagram for the complement of event E, E′, we shade the region outside the circle representing the event E.

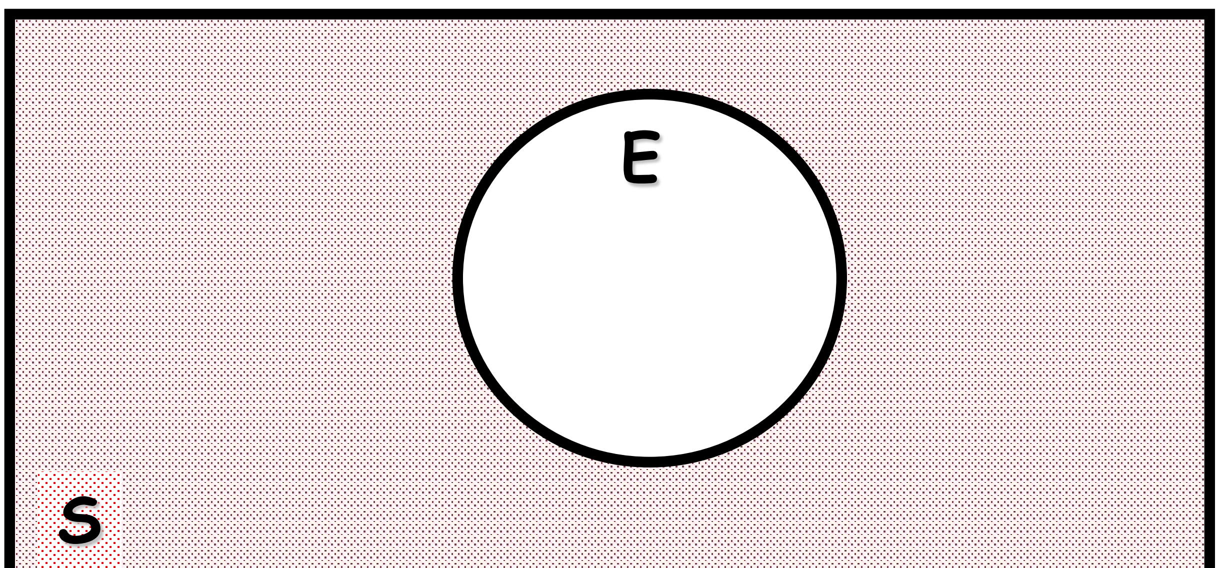
Venn Diagram for the Complement of an Event

- The complement of event E, E´.



Venn Diagram for the Complement of an Event

- The complement of event E, E´. <u>NOTE</u>: We shade the region that is *not in E* or the region *outside E*.



Example

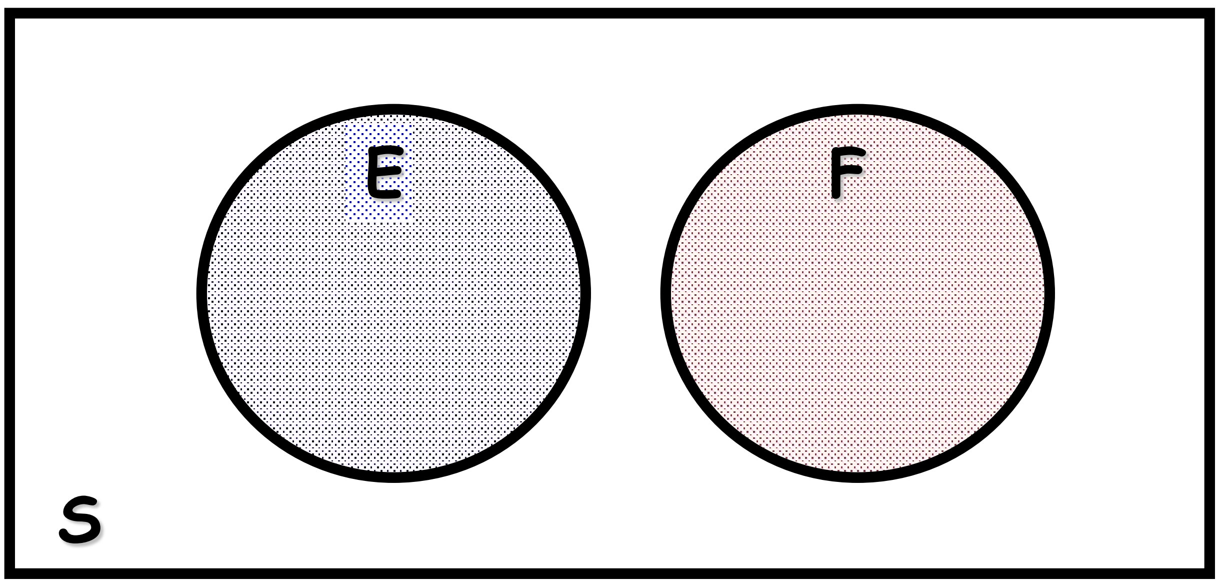
- A single die is rolled. What is the probability that the die does not display a five?

Disjoint Events

- Two events, E and F, are disjoint or mutually exclusive if they have no simple events in common.

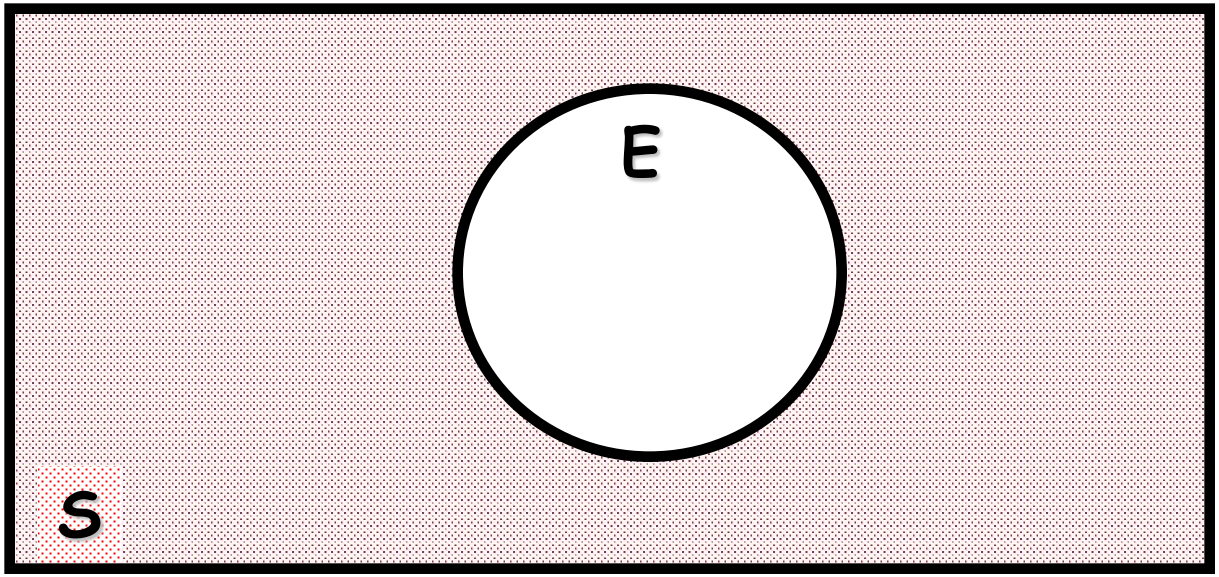
Venn Diagram for two Disjoint Events

- The two disjoint events, E and F, do not overlap.



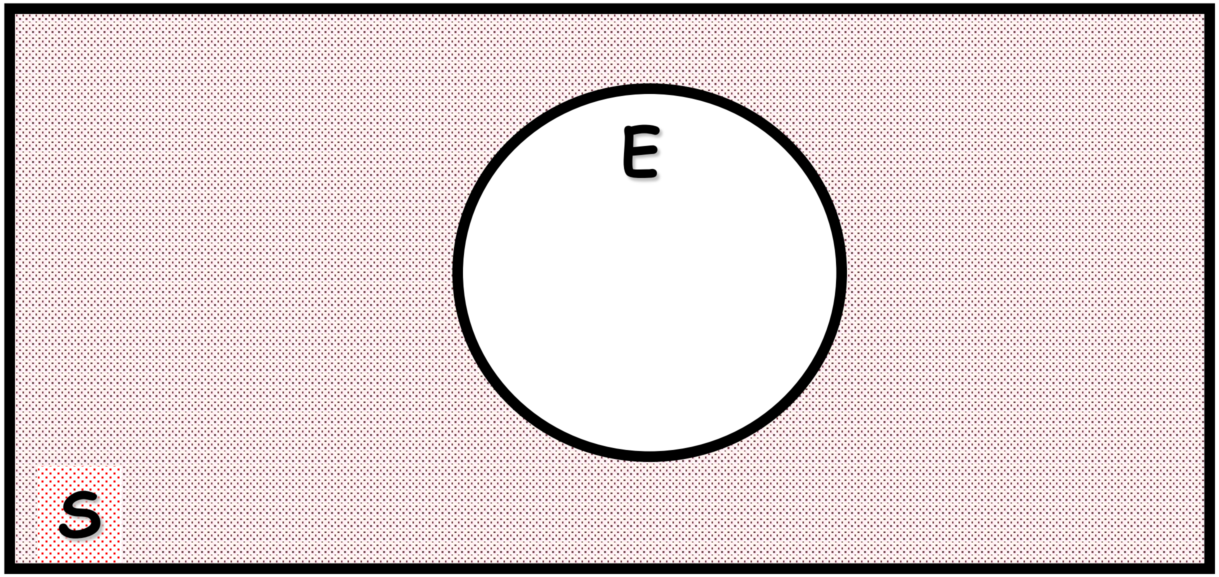
Complement of an Event

- The complement of event E, E´, and the event E are disjoint.



Complement of an Event

- The union of the event E and its complement E´ is the sample space, S.
- $E \cup E' = S$

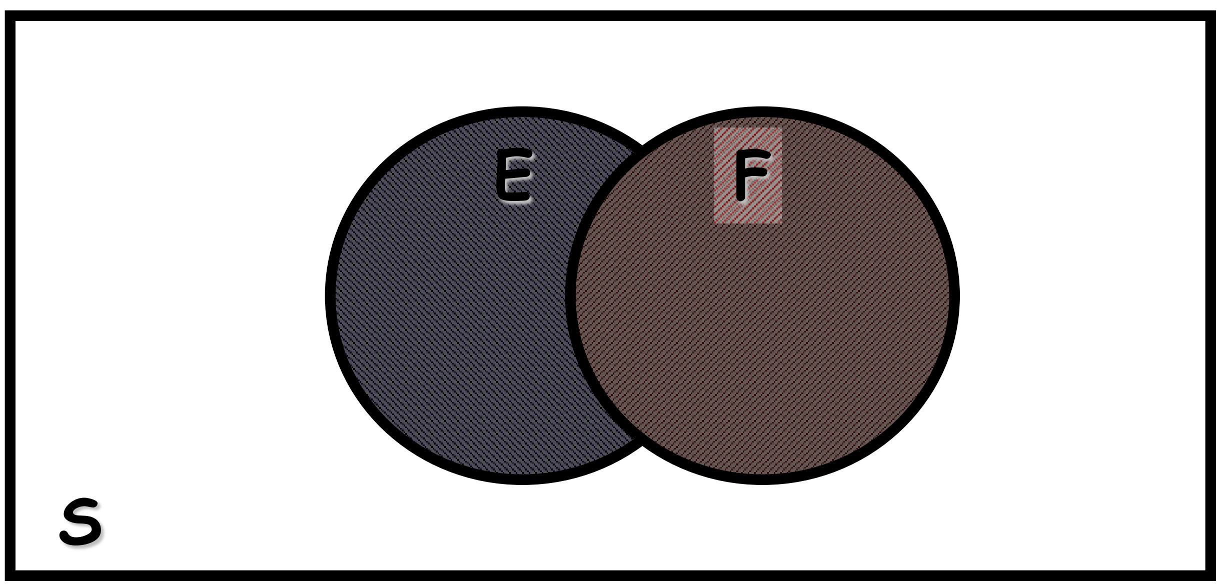


Non-Disjoint Events

- Two events, E and F, are *not* disjoint or *not* mutually exclusive if they have at least one simple event in common.

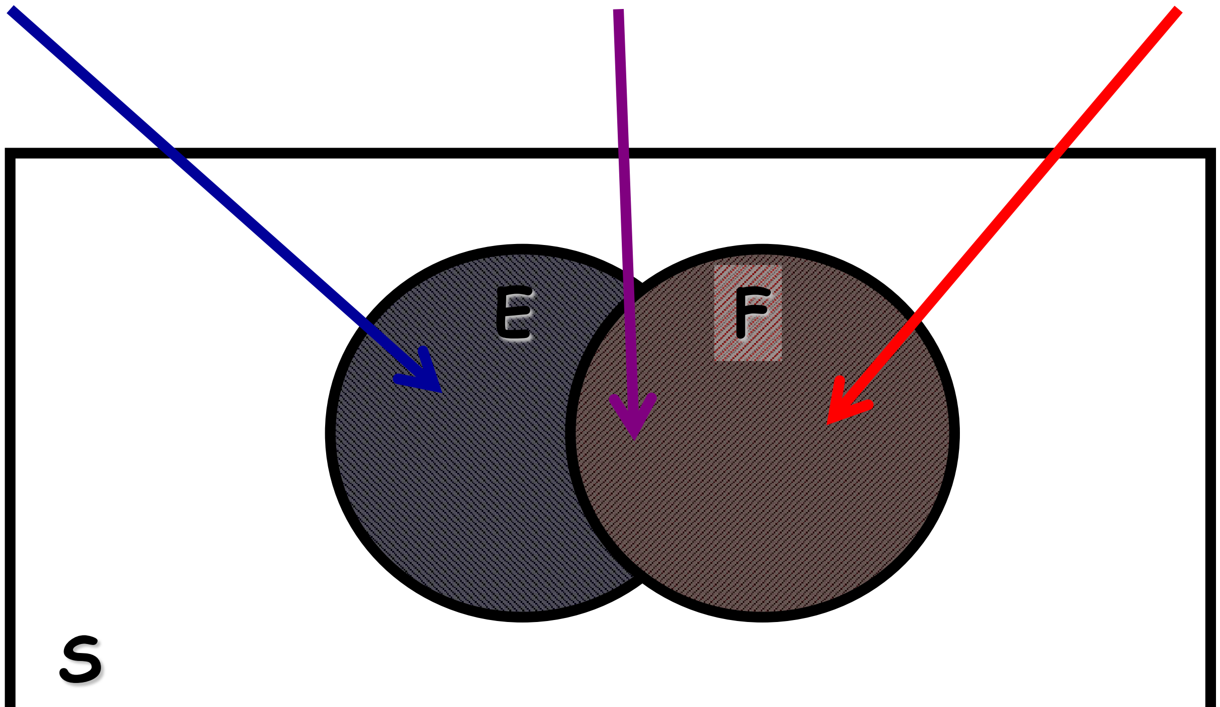
Venn Diagram for two Events that are not Disjoint

- **The two events, E and F, are *not* disjoint if they overlap.**



Venn Diagram for two Events that are not Disjoint

- In E and not in F
- In E and in F
- In F and not in E



$E \cap F$

- Given two events E and F, the event consisting of the simple events that the events E and F share or the simple events that events E and F have in common is denoted $E \cap F$.
- The event $E \cap F$ is the event that contains the simple events that are contained in *both* events E and F.

$E \cap F$

- $E \cap F$ denotes the intersection of events E and F
- The intersection of events E and F, $E \cap F$, is the set of simple events that are shared by events E and F
- The intersection of events E and F, $E \cap F$, is the set of simple events are in E *and* in F.

$E \cap F$

- $E \cap F$ denotes the *intersection* of events E and F
- The *intersection* of events E and F, $E \cap F$, is the set of simple events that are shared by events E and F
- The *intersection* of events E and F, $E \cap F$, is the set of simple events are in E *and* in F.
- KEY WORD: and

$E \cap F'$

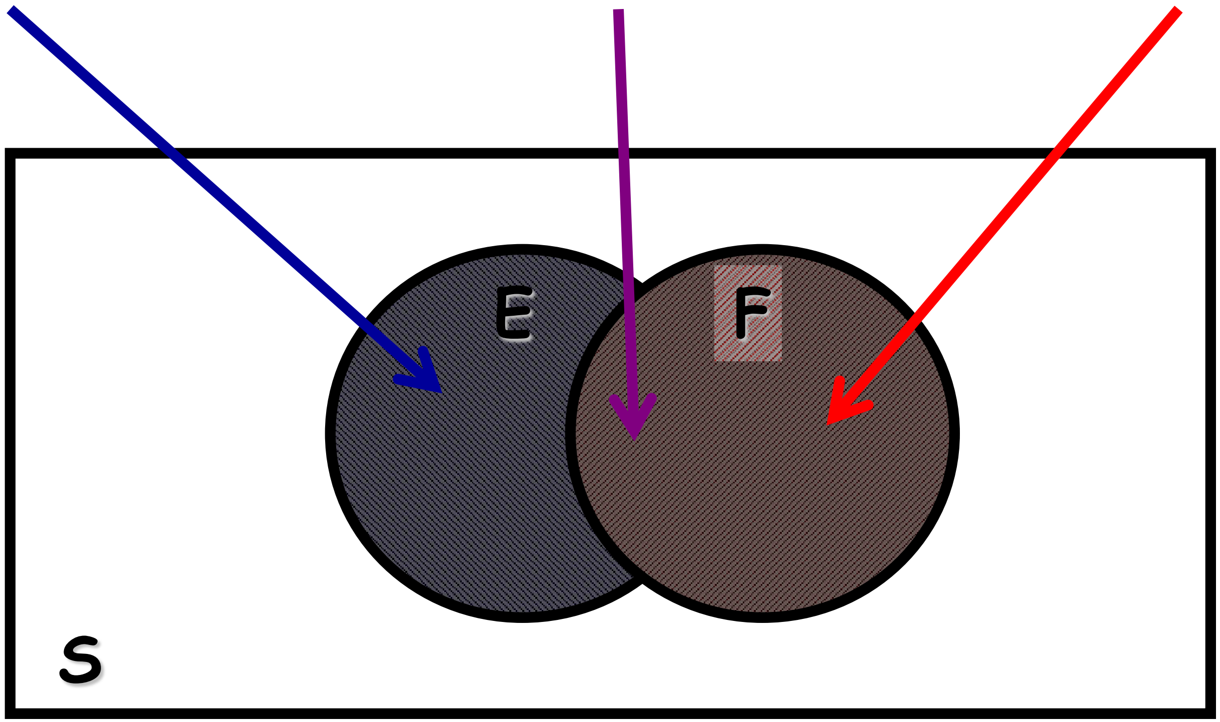
- Given two events E and F, the event consisting of the simple events that are in event E and are not in event F is denoted $E \cap F'$.
- The event $E \cap F'$ is the event that contains the simple events that are contained in *both* events E and F'.

$E' \cap F$

- Given two events E and F, the event consisting of the simple events that are not in event E and are in event F is denoted $E' \cap F$.
- The event $E' \cap F$ is the event that contains the simple events that are contained in *both* events E' and F.

Venn Diagram for two Events that are not Disjoint

- $E \cap F'$ or $F' \cap E$
- $E \cap F$
- $E' \cap F$ or $F \cap E'$



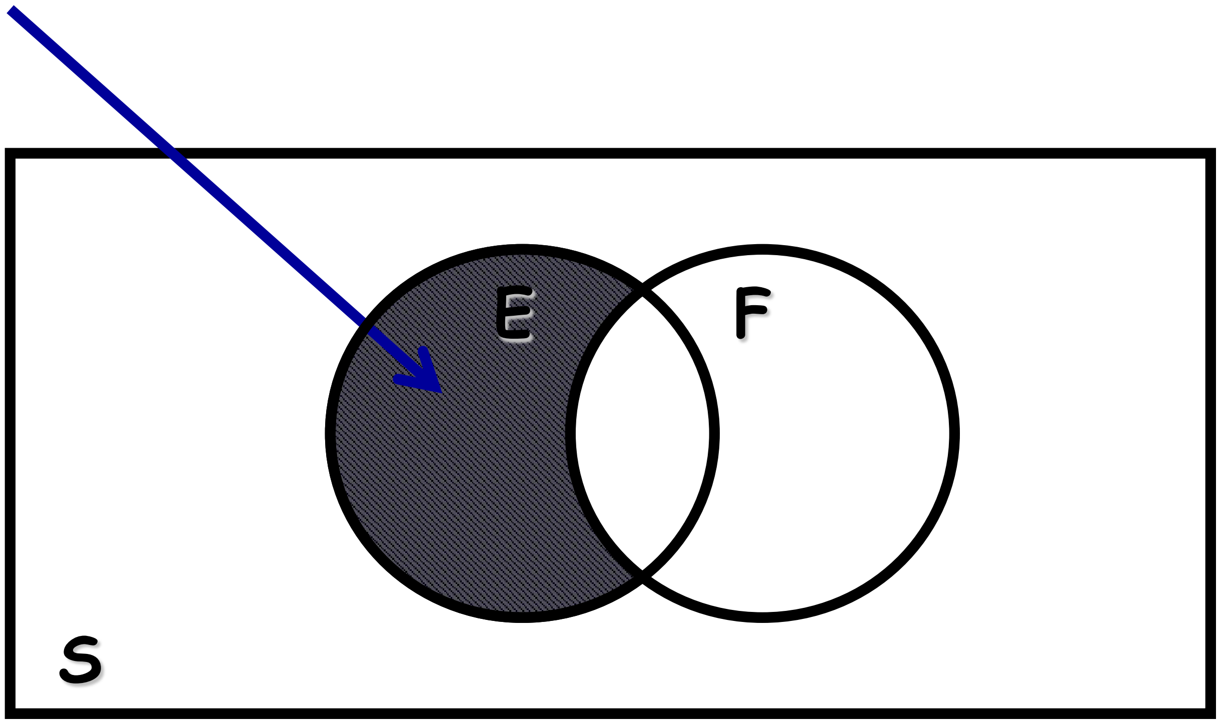
$E \cup F$

- Given two events E and F, the event consisting of the simple events that are contained in event E or contained in event F is denoted $E \cup F$.
- The event $E \cup F$ is the event that contains the simple events that are contained in event E *or* event F.

Each Part is contained in $E \cup F$ since each part is in event E *or* in event F

- $E \cap F'$
  or $F' \cap E$

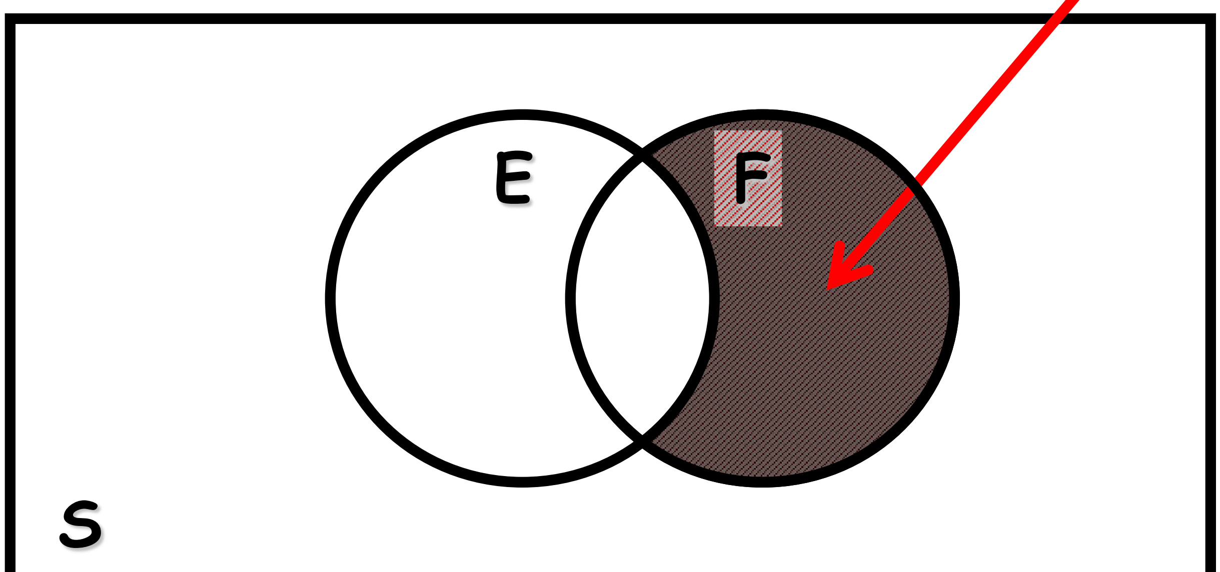
<u>Note</u>: This is the part of E that is *outside* of F



Each Part is contained in $E \cup F$ since each part is in event E *or* in event F

<u>Note</u>: This is the part of F that is *outside* E.

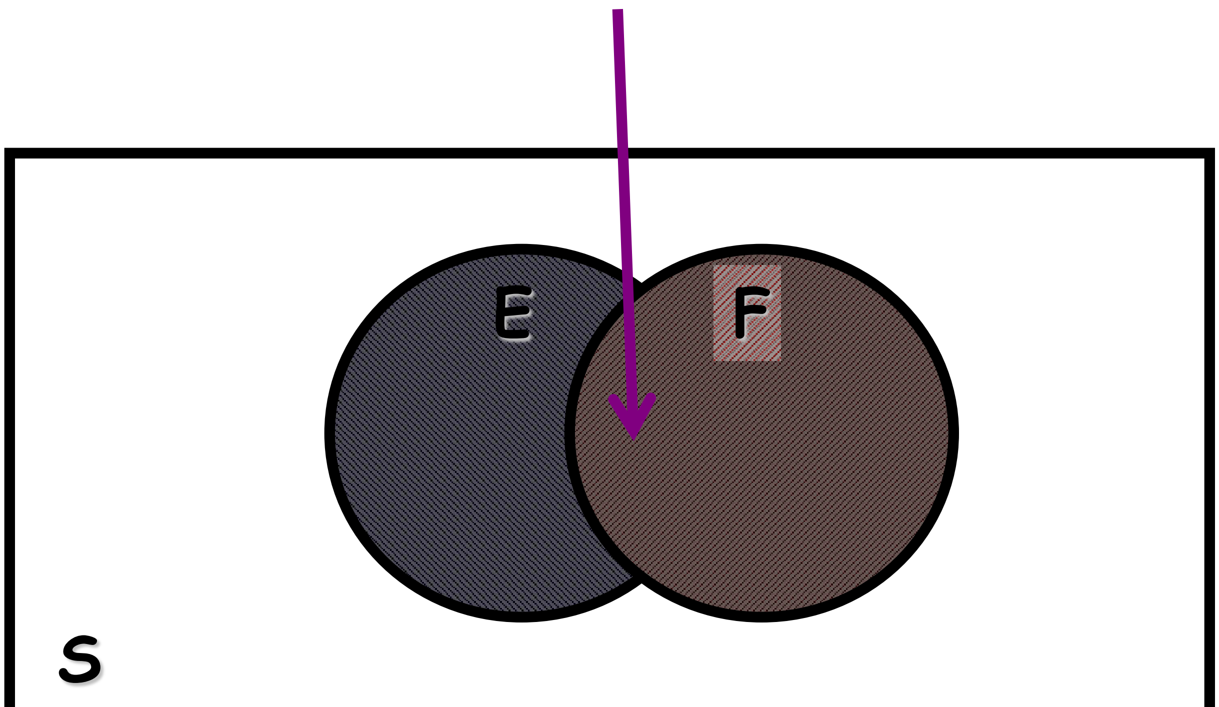
- $E' \cap F$ or $F \cap E'$



Each Part is contained in $E \cup F$ since each part is in event E *or* in event F

Note: This part is in E.

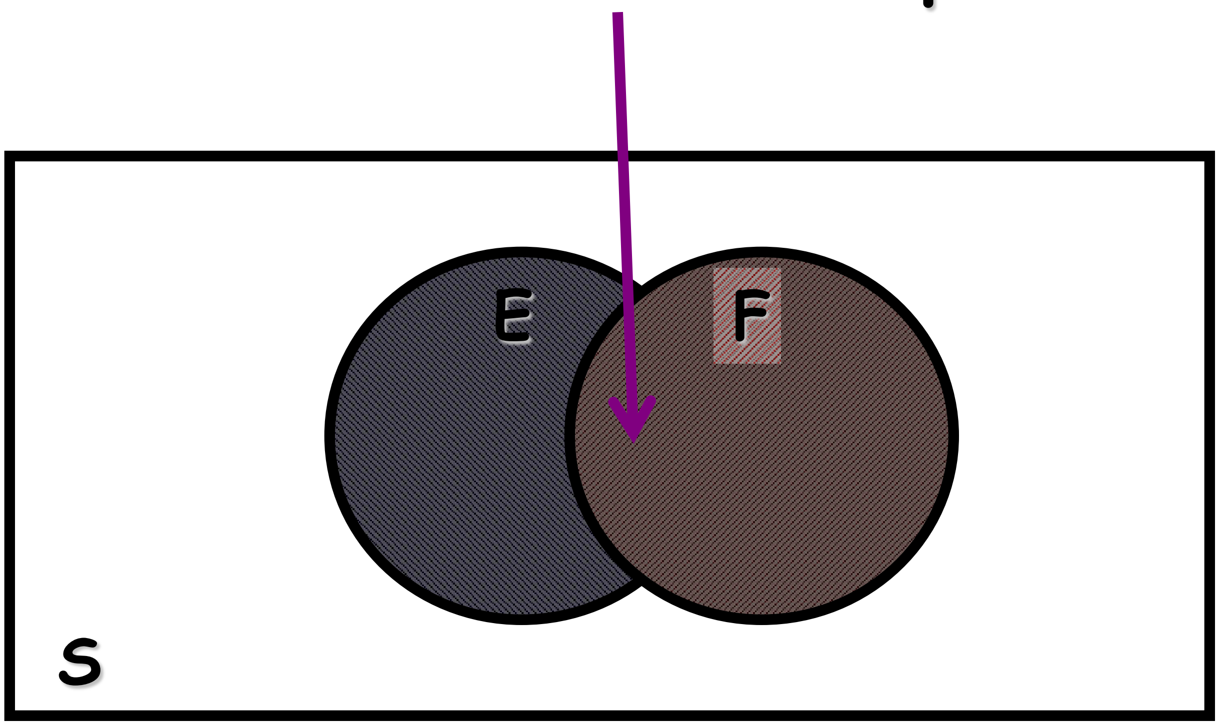
- $E \cap F$



Each Part is contained in $E \cup F$ since each part is in event E *or* in event F

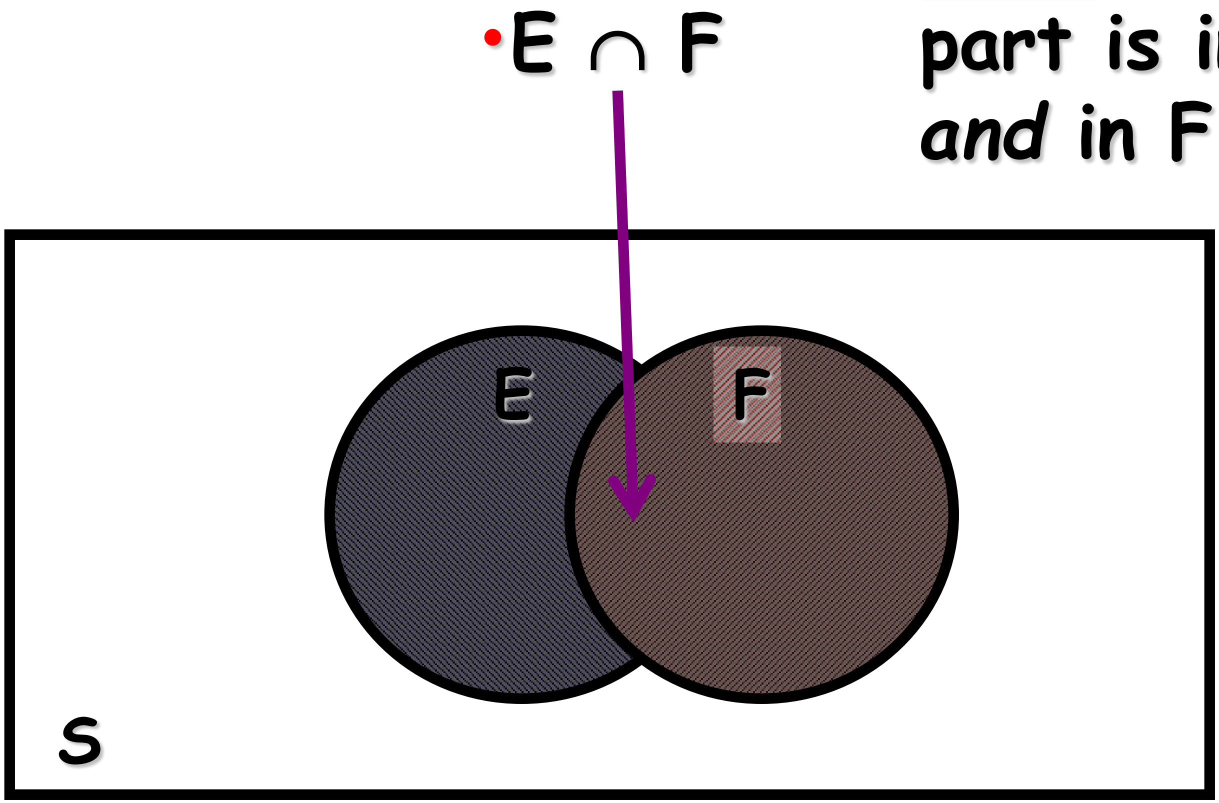
- $E \cap F$

Note: This part is in F.



Each Part is contained in $E \cup F$ since each part is in event E *or* in event F

<u>Note</u>: This part is in E *and* in F.

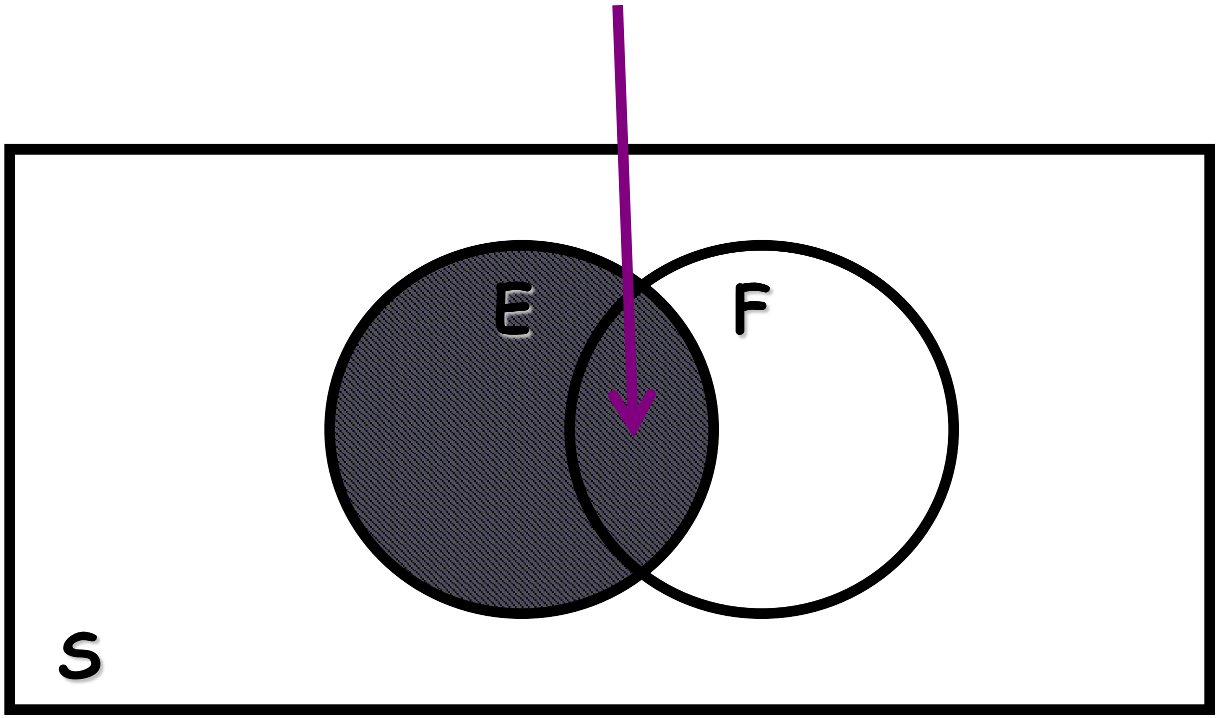


$E \cup F$

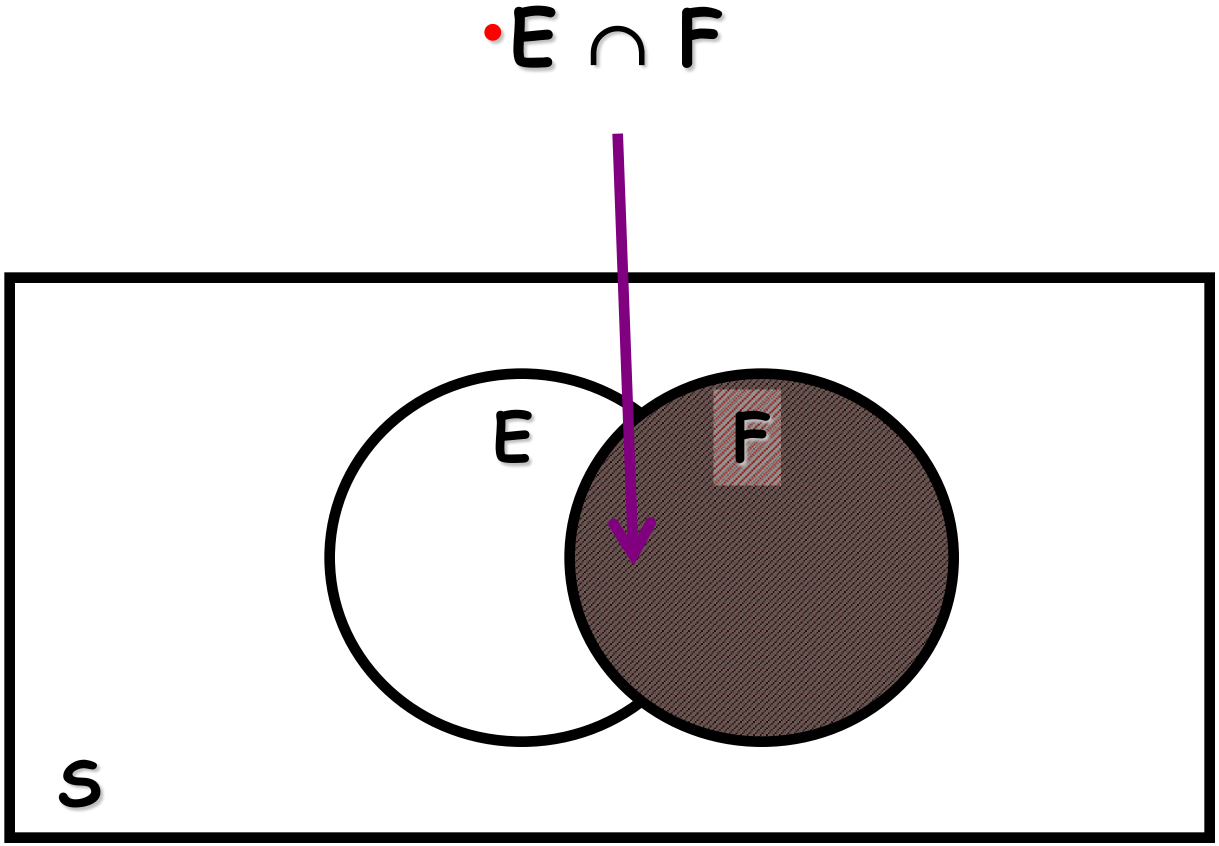
- $E \cup F$ denotes the *union* of events E and F.
- The *union* of events E and F, $E \cup F$, is the set of simple events that are in event E or in event F.
- The *union* of events E and F, $E \cup F$, is the set of simple events are event E, event F, or both event E and event F.
- KEY WORD: or

$E \cap F$ is contained in E

- $E \cap F$



E ∩ F is contained in F

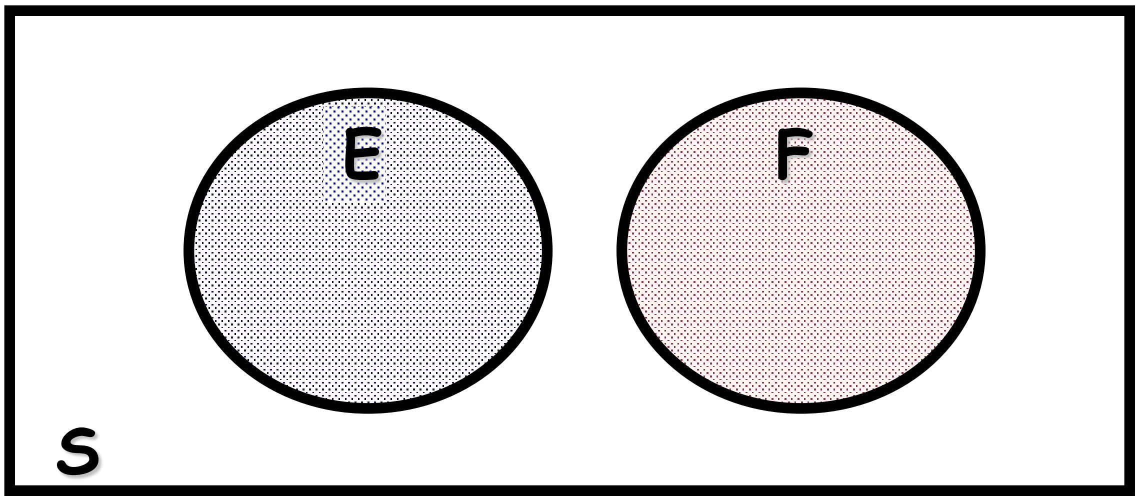


Probability of $E \cap F$

- There is no special rule for determining the probability of $E \cap F$
- To determine the $P(E \cap F)$, you must
  - Know the sample space
    - Know the number of simple events in the sample space
  - Know the event $E \cap F$
    - Know the number of simple events in the event $E \cap F$

Probability of E ∪ F

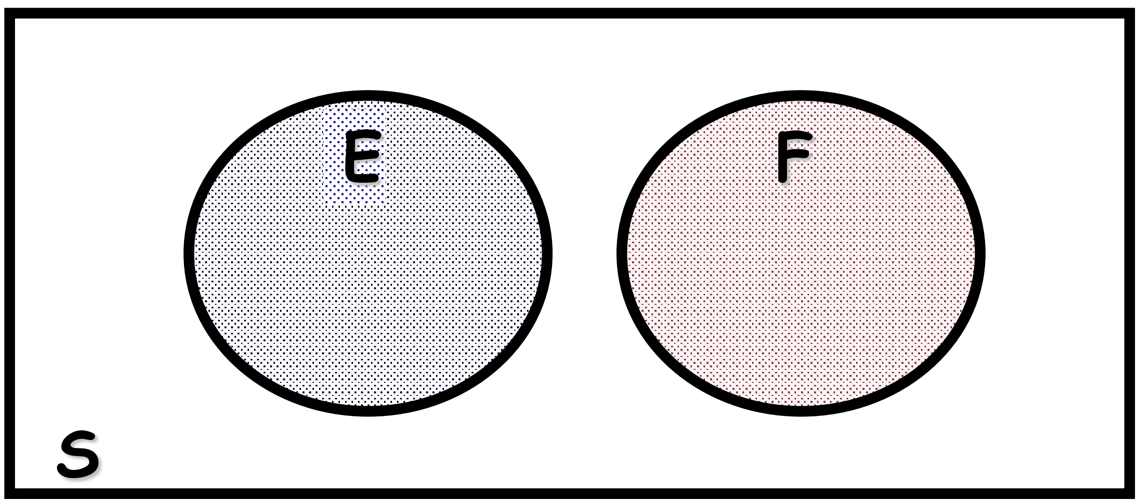
- Suppose events E and F are disjoint



- The probability that event E ∪ F occurs is the sum of the probabilities of the individual events E and F

Probability of E ∪ F

- Suppose events E and F are disjoint



- $P(E \cup F) = P(E) + P(F)$

Example

- A fair die is rolled. What is the probability that the die displays a one or a five?

Example

- A fair die is rolled. What is the probability that the die displays a one or a five?
- Since the events *the die displays a one* and *the die displays a five* are disjoint,
  P(one or five) = P(one) + P(five)
  = 1/6 + 1/6
  = 2/6
  = 1/3

Example

- A card is drawn from a fair poker deck. What is the probability that the card is an Ace or a ten?

Example

- A card is drawn from a fair poker deck. What is the probability that the card is an Ace or a ten?
- Since the events *the card is an Ace* and *the card is a ten* are disjoint,
  P(Ace or ten) = P(Ace) + P(ten)
  = 4/52 + 4/52
  = 8/52
  = 2/13

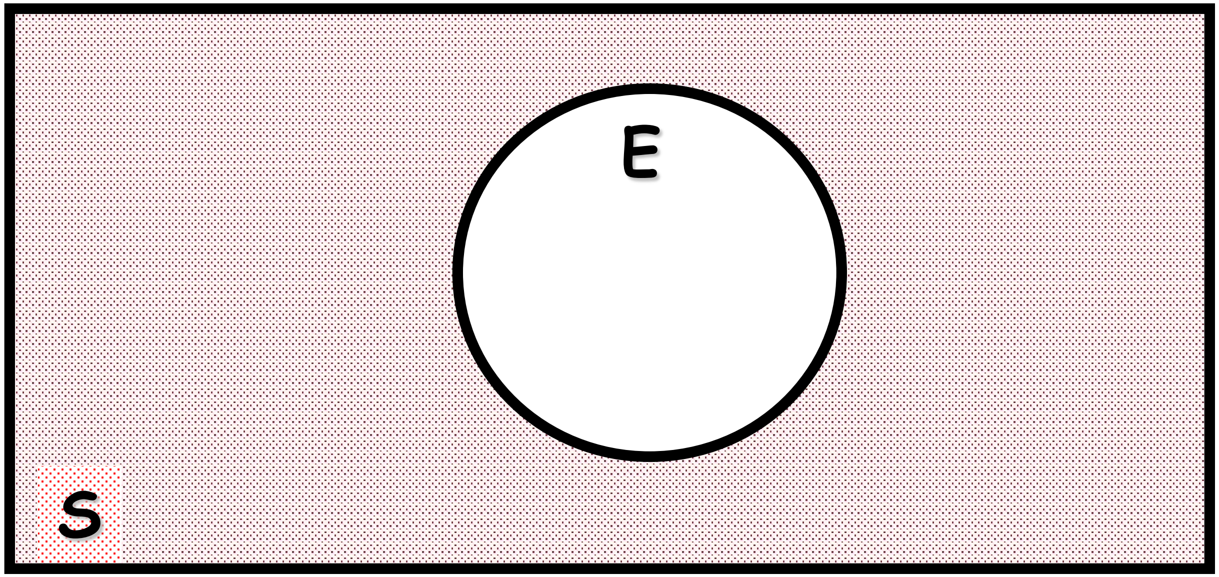
Example

- A card is drawn from a fair poker deck. What is the probability that the card is an Ace or a ten?
- Since the events *the card is an Ace* and *the card is a ten* are disjoint,
  P(Ace or ten) = P(Ace) + P(ten)
  = 1/13 + 1/13*
  = 2/13

*The fractions are simplified first.

Probability of the Complement of an Event

- $P(E) + P(E') = P(S)$
- $P(E) + P(E') = 1$
- $P(E') = 1 - P(E)$



Example

- **A fair die is rolled. What is the probability that the die does not display a five?**

Example

- A fair die is rolled. What is the probability that the die does not display a five?
- P(not five) = 1 – P(five)

  = 1 – 1/6

  = 5/6

Example

- A card is drawn from a fair poker deck. What is the probability that the card is not a face card?
- P(not face card) = 1 – P(face card)

  = 1 – 12/52

  = 40/52

  = 10/13

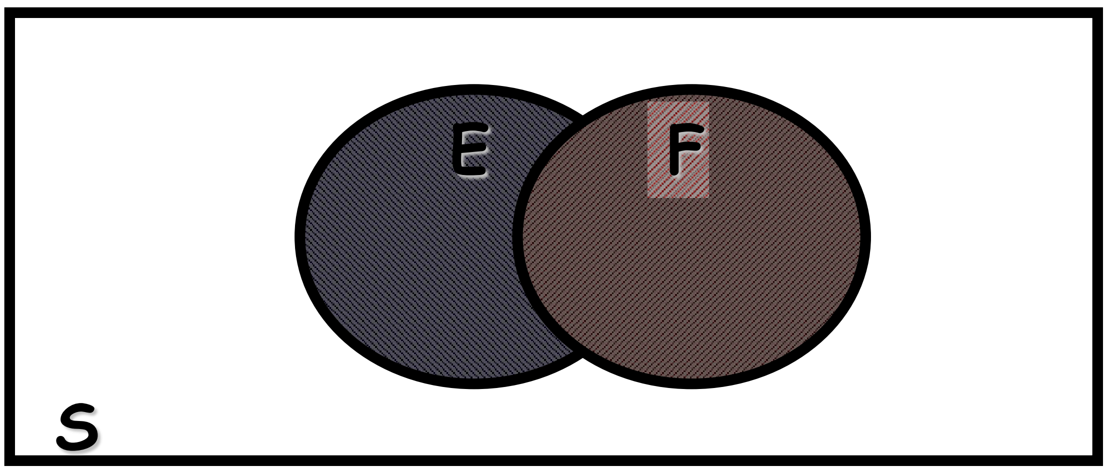
Example

- A card is drawn from a fair poker deck. What is the probability that the card is not a face card?
- P(not face card) = 1 – P(face card)
  = 1 – 3/13*
  = 10/13

*The fraction is simplified first.

Probability of $E \cup F$

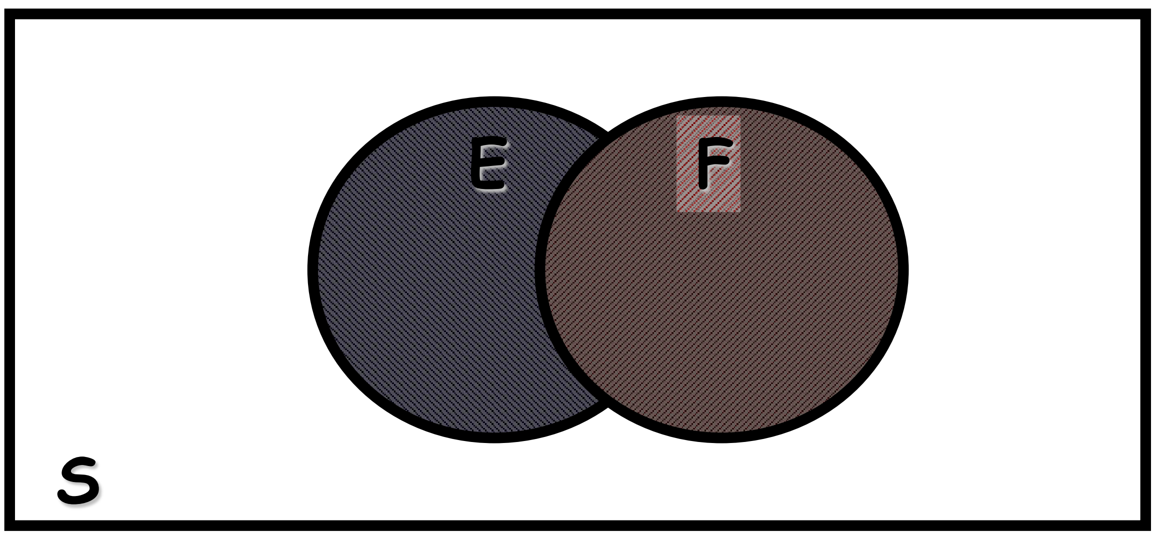
- Suppose events E and F are not disjoint



- The probability that event $E \cup F$ occurs is the sum of the probabilities of events E and F from which the probability of the intersection of event E and F, $E \cap F$, is *subtracted*

Probability of $E \cup F$

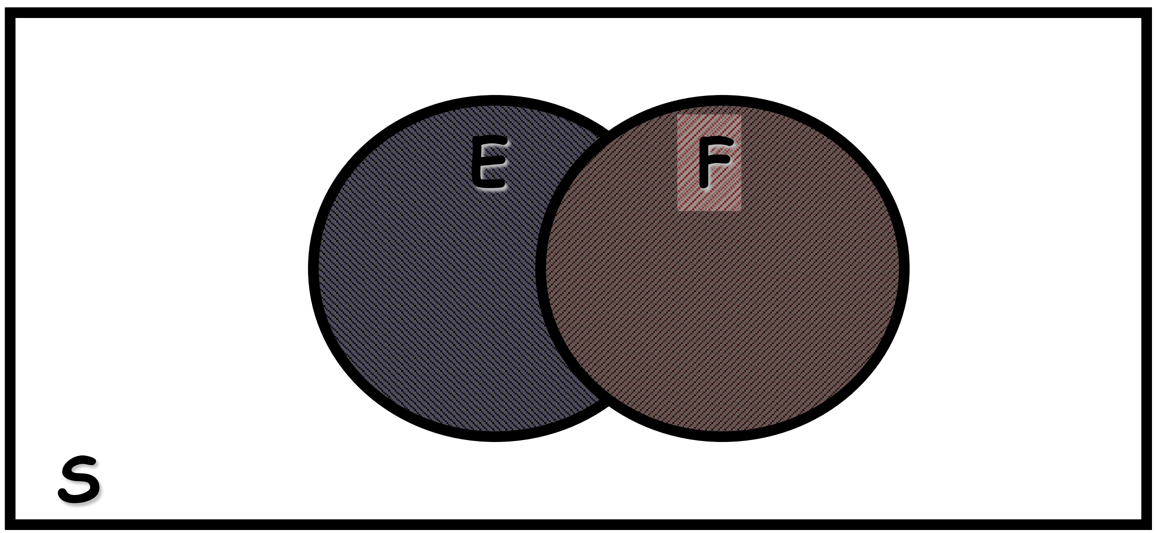
- Suppose events E and F are not disjoint



- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Probability of $E \cup F$ for non-disjoint events E and F

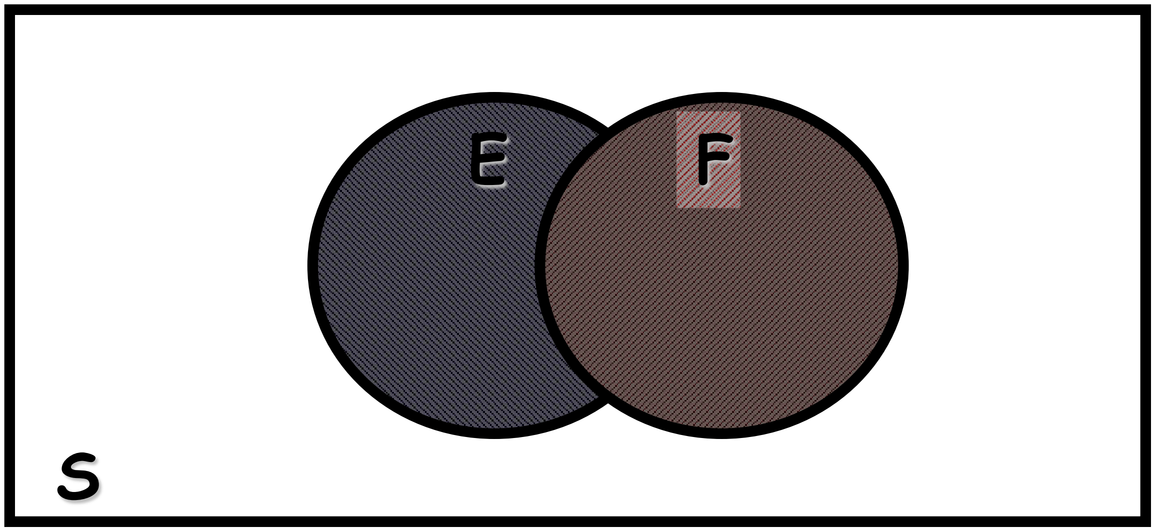
- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$



- Why subtract $P(E \cap F)$?

Probability of $E \cup F$ for non-disjoint events E and F

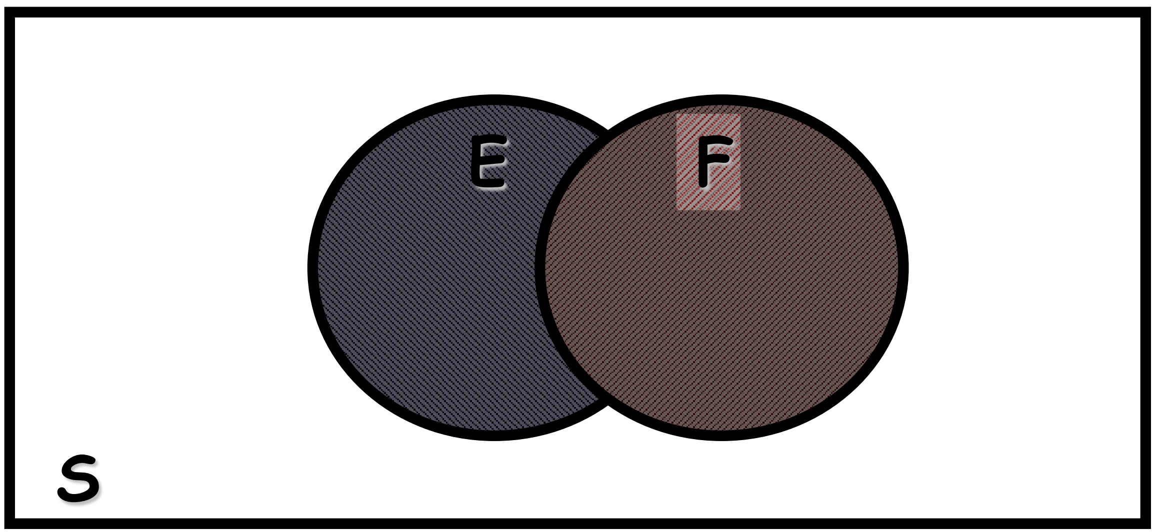
- Why subtract $P(E \cap F)$?



We subtract $P(E \cap F)$ since $E \cap F$ is contained in event E and $E \cap F$ is contained in event F.

Probability of $E \cup F$ for non-disjoint events E and F

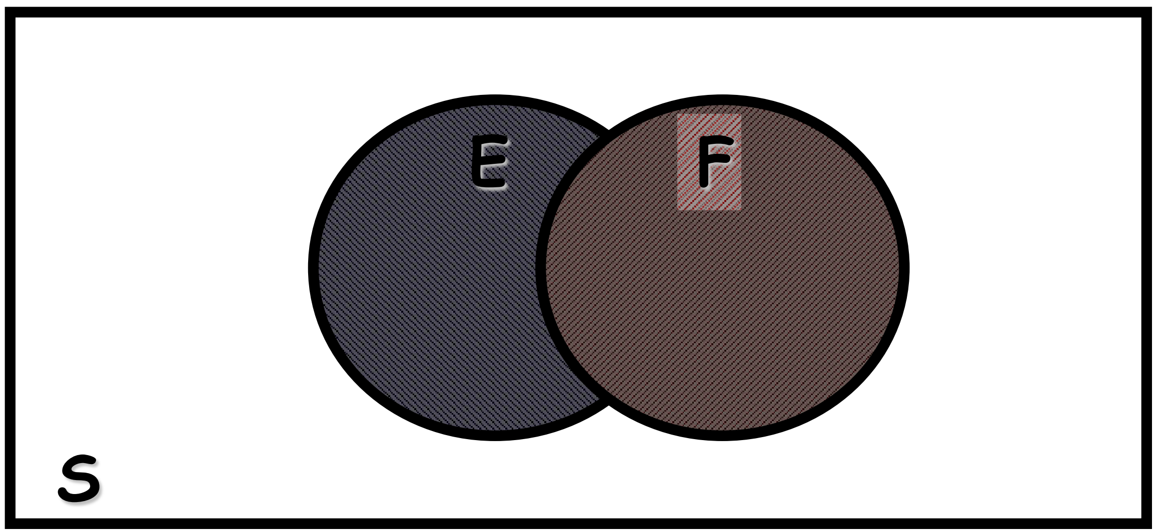
- Why subtract $P(E \cap F)$?



If we do not subtract $P(E \cap F)$ then $P(E \cup F)$ would count each of the simple events in $E \cap F$ *twice*.

Probability of $E \cup F$ for non-disjoint events E and F

- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$



- This is also called the Addition Rule.

Example

- A card is drawn from a fair poker deck. What is the probability that the card is a diamond or a face card?
- P(♦ or face card) = P(♦) + P(face card)
  - P(♦face card)
  = 13/52 + 12/52
  - 3/52
  = 22/52
  = 11/26

Example

- A card is drawn from a fair poker deck. What is the probability that the card is a diamond or a face card?
- P(♦ or face card) = P(♦) + P(face card)
  - P(♦face card)
  = 1/4 + 3/13
  - 3/52*
  = 22/52
  = 11/26

*The fractions simplified first.