Random Variables and Discrete Probability Distributions

Random Variables
and
Discrete Probability Distributions

- What is a random variable?
  - Are there different types of random variables?
- What is a discrete probability distribution?

Random Variable

- A random variable is a variable that takes on values associated with outcomes of a probability experiment.
  - The values of a random variable are numerical values/measures that are either discrete or continuous.

Random Variable

- A random variable is a variable that takes on values associated with outcomes of a probability experiment.
  - Random variables are denoted using letters such as X.

Random Variable

- A discrete random variable has either a finite number of values or a countable number of values.

Random Variable

- A discrete random variable has either a finite number of values or a countable number of values.
- Examples:
  - The number displayed on the top face of a fair die
  - The prizes associated with a raffle

Random Variable

- A discrete random variable has either a finite number of values or a countable number of values.
- Examples:
  - The number of times a fair coin lands tail side up when the coin is tossed five times

Random Variable

- A discrete random variable has either a finite number of values or a countable number of values.
- Examples:
  - The sum of the numbers displayed on the top faces of a pair of fair dice

Random Variable

- A discrete random variable has either a finite number of values or a countable number of values.
- Examples:
  - The number of customers waiting in line to be served at a Starbucks

Random Variable

- A discrete random variable has either a finite number of values or a countable number of values.
- Examples:
  - The amounts, in dollars, of the prizes associated with a winning Massachusetts Daily Number lottery ticket

Random Variable

- A discrete random variable has either a finite number of values or a countable number of values.
- Examples:
  - The number of French Fries in a small order of French Fries purchased at *Burger King*

Random Variable

- A continuous random variable has an infinite number of values.

Random Variable

- A continuous random variable has an infinite number of values.
- Examples:
  - The amount of time, in minutes, it takes a student to complete a one-hour examination

Random Variable

- A continuous random variable has an infinite number of values.
- Examples:
  - The amount of water, in ounces, in an 8-ounce bottle of *Poland Springs* bottled water

Random Variable

- A continuous random variable has an infinite number of values.
- Examples:
  - The amount of water, in gallons, that flows over Niagara Falls during one hour

Random Variable

- A continuous random variable has an infinite number of values.
- Examples:
  - The weight, in ounces, of a *Big Mac* sold at *McDonald's*

Random Variable

- A continuous random variable has an infinite number of values.
- Examples:
  - The weight, in pounds, for the amount of red seedless grapes that you buy while shopping at *Stop & Shop*

Probability Distribution for a Discrete Random Variable

- A probability distribution for a discrete random variable X is a *table*, *graph*, or *formula* that specifies the probability associated with each possible value of the random variable.

Probability Distribution for a Discrete Random Variable

- Since probabilities are between zero and one, inclusive, the values of the probabilities associated with a discrete random variable must be between zero and one, inclusive.

Probability Distribution for a Discrete Random Variable

- Since probabilities are between zero and one, inclusive, the values of the probabilities associated with a discrete random variable must be between zero and one, inclusive.
  - Letting $P(x)$ denote the probability that the value of the random variable X is equal to x, these probabilities must be such that $0 \leq P(x) \leq 1$

Probability Distribution for a Discrete Random Variable

- Since probabilities are between zero and one, inclusive, the values of the probabilities associated with a discrete random variable must be between zero and one, inclusive.
  - *Alternate*: Letting $P(X = x)$ denote the probability that the value of the random variable X is equal to x, these probabilities must be such that $0 \leq P(X = x) \leq 1$

Probability Distribution for a Discrete Random Variable

- Since all the probabilities associated with the values of the random variable are specified by the probability distribution, the sum of these probabilities must be one.
  - Letting $P(x)$ denote the probability that the value of the random variable X is equal to x, these probabilities must be such
    $\sum P(x) = 1$

Probability Distribution for a Discrete Random Variable

- Since all the probabilities associated with the values of the random variable are specified by the probability distribution, the sum of these probabilities must be one.
  - *Alternate:* Letting $P(X = x)$ denote the probability that the value of the random variable X is equal to x, these probabilities must be such
    $\sum P(X = x) = 1$

Probability Distribution for a Discrete Random Variable

- Requirements for a probability distribution for a discrete random variable
  - For $P(x)$ the probability that the value of the random variable X is equal to x, the following must be true:
    - $0 \leq P(x) \leq 1$
    - $\sum P(x) = 1$

Probability Distribution for a Discrete Random Variable

- *Alternate Notation* - Requirements for a probability distribution for a discrete random variable
  - For $P(X = x)$ the probability that the value of the random variable X is equal to x, the following must be true:
    - $0 \leq P(X = x) \leq 1$
    - $\sum P(X = x) = 1$

What is the probability distribution for the toss of one fair coin?

What is the probability distribution for the toss of one fair coin?

- It would be best to use a table to represent the probability distribution for the toss of a fair coin.

What is the probability distribution for the toss of one fair coin?

- First, we must determine the possible outcomes for tossing a fair coin.

What is the probability distribution for the toss of one fair coin?

- First, we must determine the possible outcomes for tossing a fair coin.
- The possible outcomes are for the coin to land head side up or tail side up.

What is the probability distribution for the toss of one fair coin?

- Let Heads denote the coin landing head side up.
- Let Tails denote the coin landing tail side up.

What is the probability distribution for the toss of one fair coin?

- Second, we must determine the probability associated with each outcome.

What is the probability distribution for the toss of one fair coin?

- Second, we must determine the probability associated with each outcome.
- The probability that the coin lands head side up is ½.
- The probability that the coin lands tail side up is ½.

What is the probability distribution for the toss of one fair coin?

- P(Heads) = ½
- P(Tails) = ½

- Using the alternate notation
  - P(X = Heads) = ½
  - P(X = Tails) = ½

What is the probability distribution for the toss of one fair coin?

- Finally, we set up the table. We create this table in a similar manner to the way in which we create frequency distributions and relative frequency distributions.
  - Put the values of the variable in the first column

What is the probability distribution for the toss of one fair coin?

- Finally, we set up the table. We create this table in a similar manner to the way in which we create frequency distributions and relative frequency distributions.
  - Put the corresponding probabilities in the second column

What is the probability distribution for the toss of one fair coin?

x	P(x)
Heads	½
Tails	½

What is the probability distribution for the toss of one fair coin?

x	P(x)
Heads	$\frac{1}{2}$
Tails	$\frac{1}{2}$

Don't forget the title! Otherwise, the reader will not understand what your table represents.

What is the probability distribution for the toss of one fair coin?

Probability Distribution for the Toss of a Fair Coin

x	P(x)
Heads	$\frac{1}{2}$
Tails	$\frac{1}{2}$

What is the probability distribution for the toss of one fair coin?

Using the alternate notation

Probability Distribution for the Toss of a Fair Coin

x	$P(X = x)$
Heads	$\frac{1}{2}$
Tails	$\frac{1}{2}$

What is the probability distribution for the toss of one fair coin?

- If we change the point of view from Heads and Tails to the number of Heads then the values of the random variable are numerical.
  - Heads becomes 1 since there is one Head.
  - Tails becomes 0 since there are zero Heads.

What is the probability distribution for the toss of one fair coin?

Using a different point of view -

Probability Distribution for the Number of Heads for the Toss of a Fair Coin

x	$P(x)$
1	$\frac{1}{2}$
0	$\frac{1}{2}$

What is the probability distribution for the toss of one fair coin?

Using a different point of view -

Probability Distribution for the Number of Heads for the Toss of a Fair Coin

x	P(X = x)
1	½
0	½

What is the probability distribution for the roll of one fair die?

What is the probability distribution for the roll of one fair die?

- First, we must determine the values of the random variable.

What is the probability distribution for the roll of one fair die?

- First, we must determine the values of the random variable.
  - That is, what are the possible outcomes for the roll of a fair die?

What is the probability distribution for the roll of one fair die?

- That is, what are the possible outcomes for the roll of a fair die?
  - Let the outcomes be represented by the number of dots displayed on the top face of the die when it lands.

What is the probability distribution for the roll of one fair die?

- The possible outcomes for the roll of a fair die are
  - 1
  - 2
  - 3
  - 4
  - 5
  - 6

What is the probability distribution for the roll of one fair die?

- Next, we must determine the associated probability for each value of the random variable.

What is the probability distribution for the roll of one fair die?

- Next, we must determine the associated probability for each value of the random variable.
  - That is, we must determine the associated probability for each outcome.

What is the probability distribution for the roll of one fair die?

- For this experiment, the associated probability for each outcome is 1/6.

What is the probability distribution for the roll of one fair die?

- Finally, we create the table. Again, this table is set up in a similar manner to the frequency distribution and the relative frequency distribution tables.

What is the probability distribution for the roll of one fair die?

- Finally, we create the table. Again, this table is set up in a similar manner to the frequency distribution and the relative frequency distribution tables.
  - Values of variable in first column
  - Associated probabilities in second column

What is the probability distribution for the roll of one fair die?

x	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

What is the probability distribution for the roll of one fair die?

x	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Don't forget the title! Otherwise, the reader will not understand what your table represents.

What is the probability distribution for the roll of one fair die?

Probability Distribution for the Roll of a Fair Die

x	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

What is the probability distribution for the roll of one fair die?

Using the alternate notation

Probability Distribution for the Roll of a Fair Die

x	P(X = x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Did you notice that each example meets the requirements for a discrete random variable???

Did you notice that each example meets the requirements for a discrete random variable???

What are those???

Did you notice that each example meets the requirements for a discrete random variable???

- Requirements for a probability distribution for a discrete random variable
  - For $P(x)$ the probability that the value of the random variable X is equal to x, the following must be true:
    - $0 \leq P(x) \leq 1$
    - $\sum P(x) = 1$

Requirements for a probability distribution for a discrete random variable must be met!

- If $P(x)$ is *not* such that $0 \leq P(x) \leq 1$ ***and*** $\sum P(x) = 1$ then your table, graph, or formula does *not* represent a probability distribution for a discrete random variable.

Requirements for a probability distribution for a discrete random variable must be met!

- If $P(x)$ is *not* such that $0 \le P(x) \le 1$ <u>*and*</u> $\sum P(x) = 1$ then your table, graph, or formula does *not* represent a probability distribution for a discrete random variable.

Carefully notice the "and"!! Both conditions must be met!!!

Requirements for a probability distribution for a discrete random variable must be met!

- *Alternate Notation* –
  If $P(X = x)$ is *not* such that
  $0 \leq P(X = x) \leq 1$ *and* $\sum P(X = x) = 1$ then your table, graph, or formula does *not* represent a probability distribution for a discrete random variable.

Requirements for a probability distribution for a discrete random variable must be met!

- *Alternate Notation* –
  If $P(X = x)$ is *not* such that
  $0 \leq P(X = x) \leq 1$ *and* $\sum P(X = x) = 1$ then your table, graph, or formula does *not* represent a probability distribution for a discrete random variable.

Carefully notice the "and"!! Both conditions must be met!!!

Did you notice that if the examples meet the requirements for a discrete random variable???

Did you notice that if the examples meet the requirements for a discrete random variable???

They do!

Did you notice that if the examples meet the requirements for a discrete random variable???

Check each for yourself!

Did you notice that if the examples meet the requirements for a discrete random variable???

Check each for yourself!

For each example, the individual probabilities are between zero and one, inclusive, *and* the sum of the probabilities is one.

Notation

- For clarity, we will use the *alternate notation* in the rest of these slides.
  - Reminder: $P(X = x)$ denotes the probability that the value of the random variable X is equal to x

Does the table below represent a probability distribution for a discrete random variable?

x	P(X = x)
Penny	0.13
Nickel	0.22
Dime	0.38
Quarter	-0.29
Dollar	0.56

Does the table below represent a probability distribution for a discrete random variable?

x	P(X = x)
Penny	0.13
Nickel	0.22
Dime	0.38
Quarter	-0.29
Dollar	0.56

NO! Although the sum of the values in the P(X = x) column is one, none of these values can be negative if these values are to be probabilities.

Does the table below represent a probability distribution for a discrete random variable?

x	P(X = x)
Penny	0.13
Nickel	0.22
Dime	0.38
Quarter	-0.29
Dollar	0.56

NO! Although the sum of the values in the P(X = x) column is one, none of these values can be negative if these values are to be probabilities.

Note: No title is provided since this is a generic example.

Does the table below represent a probability distribution for a discrete random variable?

x	$P(X = x)$
-2	0.25
1	0.3
3	0.4
5	0.05

Does the table below represent a probability distribution for a discrete random variable?

x	P(X = x)
-2	0.25
1	0.3
3	0.4
5	0.05

YES! The values in the P(X = x) column are each between zero and one, inclusive and the sum of these values is one.

Does the table below represent a probability distribution for a discrete random variable?

x	P(X = x)
-2	0.25
1	0.3
3	0.4
5	0.05

YES! The values in the P(X = x) column are each between zero and one, *inclusive* and the sum of these values is one.

Note: There are no limitations on the x values.

Note: No title is provided since this is a generic example.

Does the table below represent a probability distribution for a discrete random variable?

x	$P(X = x)$
2	0.23
4	0.3
6	0.4
8	0.05
10	0.01

Does the table below represent a probability distribution for a discrete random variable?

x	$P(X = x)$
2	0.23
4	0.3
6	0.4
8	0.05
10	0.01

NO! Although the values in the $P(X = x)$ column are each between zero and one, *inclusive*, the sum of these values is NOT one.

Note: No title is provided since this is a generic example.

Probability Histogram

- We construct a probability histogram in the same way in which we construct a frequency histogram or a relative frequency histogram except with probability on the vertical axis rather than frequency or relative frequency, respectively.

Probability Histogram

- As with a frequency histogram or a relative frequency histogram, the axes must be appropriately labeled and there must be an appropriate title.

Raffle Example

- A raffle has four prizes, a first prize of \$500, a second prize of \$200, two third prizes of \$50, and three fourth prizes of \$10. Suppose 1000 tickets are sold for \$1 each.

- Create the discrete probability distribution for someone who purchases one ticket for this raffle.

Raffle Example

- Although there are four prizes, there are *five* possible outcomes
  - Win First prize of $500
  - Win Second prize of $200
  - Win Third prize of $50
  - Win Fourth prize of $10
  - Not winning one of these prizes, "winning" $0

Raffle Example

- Since *not winning one of the prizes* cannot be named as a “prize”, it would be best to represent the outcomes as using the value, in dollars, for each prize.

Raffle Example

- *Since not winning one of the prizes* cannot be named as a "prize", it would be best to represent the outcomes as using the value, in dollars, for each prize.
  - The outcomes are $500, $200, $50, $10, and $0.

Raffle Example

- **Next, we determine the probability that each outcome occurs.**

Raffle Example

- Next, we determine the probability that each outcome occurs.
  - $P(X = \$500) = 1/1000$ since there is one prize of \$500 and one thousand tickets were sold.

Raffle Example

- Next, we determine the probability that each outcome occurs.
  - $P(X = \$200) = 1/1000$ since there is one prize of \$200 and one thousand tickets were sold.

Raffle Example

- Next, we determine the probability that each outcome occurs.
  - $P(X = \$50) = 2/1000$ since there are two prizes of \$50 and one thousand tickets were sold.

Raffle Example

- Next, we determine the probability that each outcome occurs.
  - That is, $P(X = \$50) = 1/500$ since there are two prizes of \$50 and one thousand tickets were sold.

Raffle Example

- Next, we determine the probability that each outcome occurs.
  - $P(X = \$10) = 3/1000$ since there are three prizes of \$10 and one thousand tickets were sold.

Raffle Example

- Next, we determine the probability that each outcome occurs.
  - P(X = \$0) = 993/1000 since there are 993 prizes of \$0, that is, there are 993 non-winning tickets, and one thousand tickets were sold.

Raffle Example

- Finally, we compile the information into a table

Raffle Example

- Finally, we compile the information into a table *that includes a meaningful title.*

Raffle Example

Probability Distribution for the Prizes, in dollars, that a Person who Buys One $1-Raffle Ticket could win in a Raffle for which One-Thousand Tickets are Sold

x	$P(X = x)$
500	1/1000
200	1/1000
50	1/500
10	3/1000
0	993/1000

Raffle Example

- In our previous analysis, we focused on the *prizes* for the raffle.
  - Point of view used – *Prize*
- Thus, our probability distribution was for the *prizes* for the raffle.

Raffle Example

- We do not need to focus on the prizes.

Raffle Example

- We could focus on the amount of money that one *wins*.

Raffle Example

- *Winnings* for a raffle are not the same as the *prizes* for the raffle.
  - *Winnings* are what you *get* in excess of what you had before you purchased the ticket.

Raffle Example

- *Winnings* for a raffle are not the same as the *prizes* for the raffle.
  - *Winnings* are what you *get* in excess of what you had before you purchased the ticket.

- Let us consider the *winnings* … .

Raffle Example

- *Alternate point of view – Winnings*

Raffle Example

- *Alternate point of view – Winnings*
  - If you win first prize and you paid $1 for your ticket, you actually *win* $499
    - Amount of Winnings
      = Amount of Prize – Ticket Price
    - First Prize Winnings = $500 - $1

Raffle Example

- *Alternate point of view – Winnings*
  - If you win second prize and you paid \$1 for your ticket, you actually *win* \$199
    - Amount of Winnings
      = Amount of Prize – Ticket Price
    - Second Prize Winnings = \$200 - \$1

Raffle Example

- *Alternate point of view – Winnings*
  - If you win third prize and you paid \$1 for your ticket, you actually *win* \$49
    - Amount of Winnings
      = Amount of Prize – Ticket Price
    - Third Prize Winnings = \$50 - \$1

Raffle Example

- *Alternate point of view – Winnings*
  - If you win fourth prize and you paid $1 for your ticket, you actually *win* $9
    - Amount of Winnings
      = Amount of Prize – Ticket Price
    - Fourth Prize Winnings = $10 - $1

Raffle Example

- *Alternate point of view – Winnings*
  - If you do not win a prize and you paid \$1 for your ticket, you actually *win* -\$1
    - Amount of Winnings
      = Amount of Prize – Ticket Price
    - Winnings for a *Losing* Ticket
      = \$0 - \$1

Raffle Example

- Combining this information into a table with an appropriate and meaningful title, we obtain the probability distribution for this raffle from a point of view of the *winnings* for someone who purchases a ticket.

Raffle Example

Probability Distribution for the Winnings, in dollars, for a Person who Buys One $1-Raffle Ticket for a Raffle for which One-Thousand Tickets are Sold

x	$P(X = x)$
499	1/1000
199	1/1000
49	1/500
9	3/1000
-1	993/1000

Raffle Example

- Note: The point of view does not affect the probabilities or the number of outcomes.
  - The point of view only affects how the values of the random variable are represented.

Raffle Example

- Note: The random variables for this example (based on the point of view) are related but not the same since

$$\text{Prize} \neq \text{Winnings.}$$

Raffle Example

- Did you notice that our probability distributions for each point of view meet the requirements for discrete probability distributions?

Raffle Example

- Did you notice that our probability distributions for each point of view meet the requirements for discrete probability distributions?
- They do! Check them for yourself!

Raffle Example

- Did you notice that our probability distributions for each point of view meet the requirements for discrete probability distributions?

- For each, our values of $P(X = x)$ are such that
  - $0 \leq P(X = x) \leq 1$ *and* $\sum P(X = x) = 1$

Expected Value for a Discrete Random Variable

Expected Value for a Discrete Random Variable

- The mean of a discrete random variable X, denoted μ_X, is determined using the formula

$$\mu_X = \sum[x \cdot P(X = x)].$$

Expected Value for a Discrete Random Variable

- The mean of a discrete random variable X, denoted μ_X, is determined using the formula

$$\mu_X = \sum[x \cdot P(X = x)].$$

- That is, to determine μ_X, we take the sum of the product of the values of the random variable, x, and their corresponding probabilities, $P(X = x)$.

Expected Value for a Discrete Random Variable

- The mean of a discrete random variable X, denoted μ_X, also referred to as the expected value.

Expected Value for a Discrete Random Variable

- The mean of a discrete random variable X, denoted μ_X, also referred to as the expected value.

Why?

Expected Value for a Discrete Random Variable

- The mean of a discrete random variable X, denoted μ_X, also referred to as the expected value.
- The mean, or expected value, also denoted E(X), is that value that we would *expect* the random variable to attain in the long run.

Expected Value for a Discrete Random Variable

- The mean of a discrete random variable X, denoted μ_X, also referred to as the expected value.
- We can think of $\mu_X = E(X)$ as the mean value of a very large (infinite, if we could) number of repetitions of the experiment in which the values of X occur in proportions equivalent to the probabilities of X.

Expected Value for a Discrete Random Variable

- We can think of $\mu_X = E(X)$ as the average value of the random variable that we would expect to obtain if we perform the experiment repeatedly.
  - Letting x_k represent the outcome of the k^{th} repetition of the experiment, $[x_1 + x_2 + \ldots + x_k + \ldots + x_n]/n$ gets closer to $\mu_X = E(X)$ as the number of repetitions of the experiment, n, increases in value.

How many Heads should we expect if we toss two fair coins?

How many Heads should we expect if we toss two fair coins?

- First, we must determine the sample space for the experiment.
  - HH
  - HT
  - TH
  - TT

How many Heads should we expect if we toss two fair coins?

- Next, we count the outcomes.
  - The outcome two Heads, HH, occurs once.
  - The outcome one Head, HT and TH, occurs twice.
  - The outcome zero Heads, TT, occurs once.

How many Heads should we expect if we toss two fair coins?

- Next, we create the probability distribution.

How many Heads should we expect if we toss two fair coins?

Probability Distribution for the Toss of Two Fair Coins

x	$P(X = x)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

How many Heads should we expect if we toss two fair coins?

- Finally, having determined the probability distribution, we can calculate the expected number of Heads, that is, the expected value for the number of Heads.

$$\mu_X = E(X) = 0 \cdot \left(\tfrac{1}{4}\right) + 1 \cdot \left(\tfrac{1}{2}\right) + 2 \cdot \left(\tfrac{1}{4}\right)$$

$$= \tfrac{1}{2} + \tfrac{1}{2}$$

$$= 1$$

How many Heads should we expect if we toss two fair coins?

- $\mu_X = E(X) = 0 \cdot (\frac{1}{4}) + 1 \cdot (\frac{1}{2}) + 2 \cdot (\frac{1}{4})$

  $= \frac{1}{2} + \frac{1}{2}$

  $= 1$
- We can expect one Head if we toss two fair coins.

What is the expected value for the roll of a fair die?

What is the expected value for the roll of a fair die?

- We have already created the probability distribution for the roll of a fair die.

What is the expected value for the roll of a fair die?

Probability Distribution for the Roll of a Fair Die

x	$P(X = x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

What is the expected value for the roll of a fair die?

- So, we calculate the expected value by taking the sum of the products of the values of the random variable and their corresponding probabilities.

What is the expected value for the roll of a fair die?

- $\mu_X = E(X) = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$
  $= (1 + 2 + 3 + 4 + 5 + 6)/6$
  $= 21/6$
  $= 3.5$

What is the expected value for the roll of a fair die?

- $\mu_X = E(X) = 1(1/6) + 2(1/6) + 3(1/6)$
  $+ 4(1/6) + 5(1/6) + 6(1/6)$
  $= (1 + 2 + 3 + 4 + 5 + 6)/6$
  $= 21/6$
  $= 3.5$
- Notice that the expected value is not one of the possible outcomes for the random variable.

What is the expected prize for our raffle example?

What is the expected prize for our raffle example?

- We have already created the probability distribution for our raffle.

What is the expected prize for our raffle example?

Probability Distribution for the Prizes, in dollars, that a Person who Buys One $1-Raffle Ticket could win in a Raffle for which One-Thousand Tickets are Sold

x	P(X = x)
500	1/1000
200	1/1000
50	1/500
10	3/1000
0	993/1000

What is the expected prize for our raffle example?

- Using our probability distribution, we can calculate the expected prize for someone purchasing one ticket for our raffle.

What is the expected prize for our raffle example?

- $\mu_X = E(X) = 500(1/1000) + 200(1/1000)$
  $+ 50(2/1000) + 10(3/1000)$
  $+ 0(993/1000)$
  $= (500 + 200 + 100 + 30)/1000$
  $= 830/1000$
  $= 0.83$
- The expected prize for someone buying a ticket for our raffle is $0.83.

What are the expected winnings for our raffle example?

What are the expected winnings for our raffle example?

- We have already created the probability distribution for our raffle.

What are the expected winnings for our raffle example?

Probability Distribution for the Winnings, in dollars, for a Person who Buys One $1-Raffle Ticket for a Raffle for which One-Thousand Tickets are Sold

x	P(X = x)
499	1/1000
199	1/1000
49	1/500
9	3/1000
-1	993/1000

What are the expected winnings for our raffle example?

- Using our probability distribution, we can calculate the expected winnings for someone purchasing one ticket for our raffle.

What are the expected winnings for our raffle example?

- $\mu_X = E(X) = 499(1/1000) + 199(1/1000)$
  $+ 49(2/1000) + 9(3/1000)$
  $+ (-1)(993/1000)$
  $= (499 + 199 + 98$
  $+ 27 - 993)/1000$
  $= -170/1000$
  $= -0.17$
- The expected winnings for someone buying a ticket for our raffle is -$0.17.

What are the expected winnings for our raffle example?

- $\mu_X = E(X) = 499(1/1000) + 199(1/1000)$
  $+ 49(2/1000) + 9(3/1000)$
  $+ (-1)(993/1000)$
  $= (499 + 199 + 98$
  $+ 27 - 993)/1000$
  $= -170/1000$
  $= -0.17$
- The expected winnings for someone buying a ticket for our raffle is -\$0.17. ***WHAT?***

What are the expected winnings for our raffle example?

- $\mu_X = E(X) = 499(1/1000) + 199(1/1000)$
  $+ 49(2/1000) + 9(3/1000)$
  $+ (-1)(993/1000)$
  $= (499 + 199 + 98$
  $+ 27 - 993)/1000$
  $= -170/1000$
  $= -0.17$
- Someone buying a ticket for our raffle can expect to lose $0.17.

For our raffle example, can we use our expected prize calculation to determine the expected winnings?

For our raffle example, can we use our expected prize calculation to determine the expected winnings?

- Yes, we can.

For our raffle example, can we use our expected prize calculation to determine the expected winnings?

- Since we know that the expected prize for someone buying a ticket for our raffle is $0.83, all we have to do is take into account her/his having paid $1 for the ticket.

For our raffle example, can we use our expected prize calculation to determine the expected winnings?

- Since we know that the expected prize for someone buying a ticket for our raffle is $0.83, all we have to do is take into account her/his having paid $1 for the ticket.
  - Recall:

Winnings = Value of Prize – Cost of Ticket

For our raffle example, can we use our expected prize calculation to determine the expected winnings?

- Since we know that the expected prize for someone buying a ticket for our raffle is \$0.83, all we have to do is take into account her/his having paid \$1 for the ticket.
- Expected Winnings = Expected Prize - \$1
  = 0.83 – 1
  = -0.17

For our raffle example, can we use our expected prize calculation to determine the expected winnings?

- Expected Winnings = Expected Prize - \$1
  = 0.83 – 1
  = -0.17
- Again, we find that someone buying a ticket for our raffle can expect to lose \$0.17.

Variance and Standard Deviation for a Discrete Random Variable

Variance and Standard Deviation for a Discrete Random Variable

- The variance of a discrete random variable X, denoted $\sigma^2_X = Var(X)$, is determined using the formula

$$\sigma^2_X = Var(X) = \sum[(x - \mu_X)^2 \cdot P(X = x)]$$
$$= \sum[x^2 \cdot P(X = x)] - (\mu_X)^2$$

- The standard deviation of a discrete random variable X, denoted σ_X, is the square root of the variance.

What is the standard deviation for the number displayed on the top face of a fair die?

What is the standard deviation for the number displayed on the top face of a fair die?

- We have already created the probability distribution for the roll of a fair die.

What is the standard deviation for the number displayed on the top face of a fair die?

Probability Distribution for the Roll of a Fair Die

x	P(X = x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

What is the standard deviation for the number displayed on the top face of a fair die?

- We have already calculated the expected value.
- Recall:
  - $\mu_X = E(X) = 1(1/6) + 2(1/6) + 3(1/6)$
    $+ 4(1/6) + 5(1/6) + 6(1/6)$
    $= (1 + 2 + 3 + 4 + 5 + 6)/6$
    $= 21/6$
    $= 3.5$

What is the standard deviation for the number displayed on the top face of a fair die?

- So, we can calculate the variance,
  - $\sigma^2_X = (1-3.5)^2(1/6) + (2-3.5)^2(1/6)$
    $+ (3-3.5)^2(1/6) + (4-3.5)^2(1/6)$
    $+ (5-3.5)^2(1/6) + (6-3.5)^2(1/6)$
    $= (-2.5)^2(1/6) + (-1.5)^2(1/6)$
    $+ (-0.5)^2(1/6) + (0.5)^2(1/6)$
    $+ (1.5)^2(1/6) + (2.5)^2(1/6)$
    $= 2(6.25 + 2.25 + 0.25)/6$
    $= 35/12$

What is the standard deviation for the number displayed on the top face of a fair die?

- Then, taking the square root of the variance, we determine the standard deviation,
  - $\sigma_X = \sqrt{(35/12)}$
    ≈ 1.707825228
    ≈ 1.707

What is the standard deviation for the number displayed on the top face of a fair die?

- Or, using the other variance formula,
  - $$\sigma^2_X = [1^2(1/6) + 2^2(1/6) + 3^2(1/6) + 4^2(1/6) + 5^2(1/6) + 6^2(1/6)] - (3.5)^2$$
    $$= [(1 + 4 + 9 + 16 + 25 + 36)/6] - 49/4$$
    $$= 91/6 - 49/4$$
    $$= (182 - 147)/12$$
    $$= 35/12$$

What is the standard deviation for the number displayed on the top face of a fair die?

- Again, the standard deviation is
  - $\sigma_X = \sqrt{(35/12)}$
    ≈ 1.707825128
    ≈ 1.707

What is the standard deviation for the number of Heads for the toss of two fair coins?

What is the standard deviation for the number of Heads for the toss of two fair coins?

- We have already created the probability distribution for the toss of two fair coins.

What is the standard deviation for the number of Heads for the toss of two fair coins?

Probability Distribution for the Toss of Two Fair Coins

x	$P(X = x)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

What is the standard deviation for the number of Heads for the toss of two fair coins?

- We have already determined the expected number of heads for the toss of two fair coins.
- Recall:
  - $\mu_X = E(X) = 0 \cdot (\frac{1}{4}) + 1 \cdot (\frac{1}{2}) + 2 \cdot (\frac{1}{4})$

    $= \frac{1}{2} + \frac{1}{2}$

    $= 1$

What is the standard deviation for the number of Heads for the toss of two fair coins?

- So, we can calculate the standard deviation.
  - $\sigma^2_X = (0-1)^2 \cdot (\frac{1}{4}) + (1-1)^2 \cdot (\frac{1}{2}) + (2-1)^2 \cdot (\frac{1}{4})$
    $= \frac{1}{4} + \frac{1}{4}$
    $= \frac{1}{2}$
  - $\sigma_X = \sqrt{(\frac{1}{2})}$
    ≈ 0.707 Heads

What is the standard deviation for the number of Heads for the toss of two fair coins?

- Or, using the other formula,

What is the standard deviation for the number of Heads for the toss of two fair coins?

- we can calculate the standard deviation, again.
  - $\sigma^2{}_X = [(0)^2 \cdot (\frac{1}{4}) + (1)^2 \cdot (\frac{1}{2}) + (2)^2 \cdot (\frac{1}{4})] - (1)^2$
    $= [\frac{1}{2} + 1] - 1$
    $= \frac{1}{2}$
  - $\sigma_X = \sqrt{(\frac{1}{2})}$
    ≈ 0.707 Heads

Variance and Standard Deviation for a Discrete Random Variable

- Note: The second formula,

$$\sigma^2_X = Var(X) = \sum[x^2 \cdot P(X = x)] - (\mu_X)^2$$

is easier/quicker to use than the first,

$$\sigma^2_X = Var(X) = \sum[(x - \mu_X)^2 \cdot P(X = x)]$$

However, it does not matter which one we use.

Variance and Standard Deviation for a Discrete Random Variable

- Note: The second formula,

  $\sigma^2_X = Var(X) = \sum[x^2 \cdot P(X = x)] - (\mu_X)^2$

  is easier/quicker to use than the first,

  $\sigma^2_X = Var(X) = \sum[(x - \mu_X)^2 \cdot P(X = x)]$

  However, it does not matter which one we use.

It is your choice.