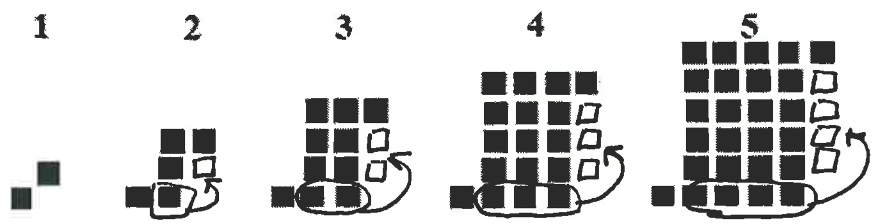
Argument #1

Problem #2: Growing S's

Question 1. Consider the pattern below. How many square tiles would there be in the eighth step of this pattern?



2 (2x2)+1 (3x3)+1 (4x4)+1 (5x5)+1

In the eighth step of this pattern, there are 65 square tiles

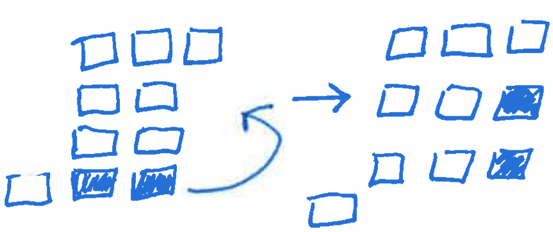
Question 2. Write an expression for the total number of square tiles in the figure at an arbitrary step (n) of the pattern.

$n^2 + 1$

Question 3. Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers, and/or symbols for your proof.

See the drawing above.

For example, in the third step of this pattern I move 2 square tiles of the bottom row to the third column of the pattern. Now we get a big square containing 9 square tiles (3x3) and 1 extra tile. The total is 10 square tiles.

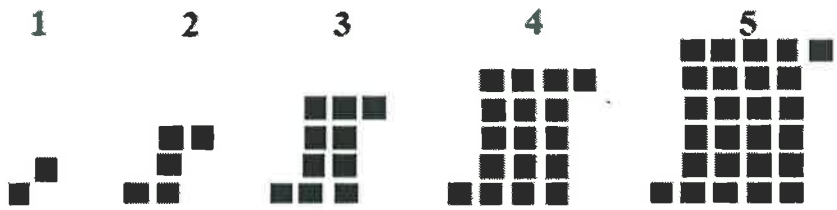


We do the same for step n of the pattern and we will get $n \times n + 1$ or (n^2+1) tiles.

Argument #2

Problem #2: Growing S's

Question 1. Consider the pattern below. How many square tiles would there be in the eighth step of this pattern?



In this pattern, the number of square tiles in the eighth step would be equal to 65.

Question 2. Write an expression for the total number of square tiles in the figure at an arbitrary step (n) of the pattern.

Let T be the total number of tiles and n be the step in the series. Then the series can be represented as:

$$T = n^2 + 1$$

Question 3. Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers, and/or symbols for your proof.

In each step of the equation, we are given (n+1) rows and (n+1) columns. From the first and last columns, we are missing n number of squares. From this, we get the equation: $T = (n+1)(n+1) - 2n$

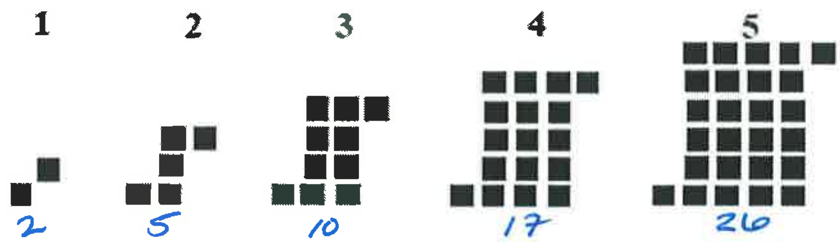
This means that for each step, we get a square made up of (n+1) rows and columns missing n unit squares from each side of the bigger square, which is representative of the pattern shown above. By simplifying the equation we can see $T = (n+1)(n+1) - 2n = n^2 + 2n + 1 - 2n = n^2 + 2n - 2n + 1 = n^2 + 1$.

Therefore, as long as the pattern holds, the equation can be represented by: $T = n^2 + 1$.

Argument #3

Problem #2: Growing S's

Question 1. Consider the pattern below. How many square tiles would there be in the eighth step of this pattern?



There would be 65 tiles in the 8th step.

Question 2. Write an expression for the total number of square tiles in the figure at an arbitrary step (n) of the pattern.

$n \times n + 1$

Question 3. Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers, and/or symbols for your proof.

Proof: Show that $(n \times n)+1$ is a valid representation of the previous pattern shown.

Let n be the arbitrary step in the pattern. Let S equal the total number of square tiles in the pattern, then

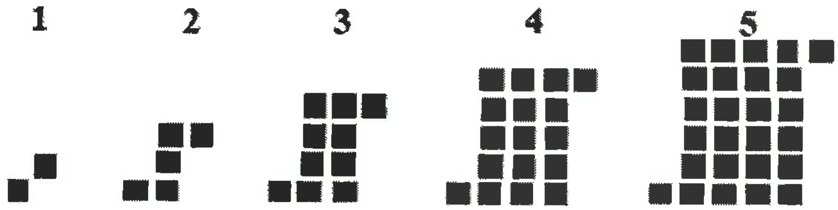
$$(n \times n)+1 = S$$

So let n be the 15th step, so you have $15 \times 15 = 225$ and according to our expression you add 1 to that which gives you 226, which equals the total number of tiles.

Argument #4

Problem #2: Growing S's

Question 1. Consider the pattern below. How many square tiles would there be in the eighth step of this pattern?



$8^2+1=65$

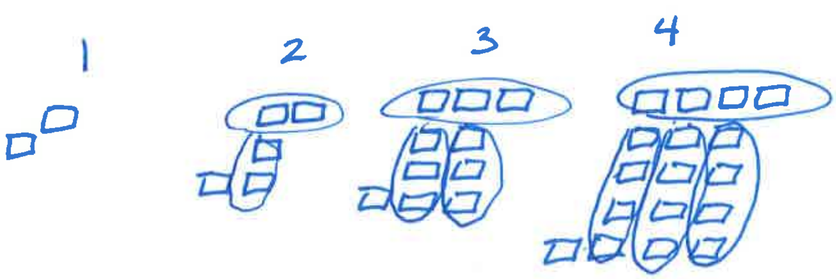
2 5 10 17 26

1^2+1 2^2+1 3^2+1 4^2+1 5^2+1

Question 2. Write an expression for the total number of square tiles in the figure at an arbitrary step (n) of the pattern.

n^2+1

Question 3. Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers, and/or symbols for your proof.



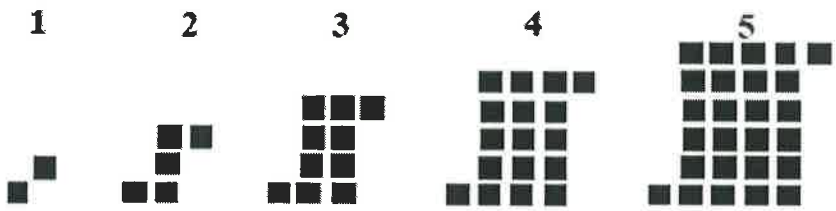
For this pattern, we always have the same number of sets and number of tiles plus one in each next step. So for step two we have 2 sets of 2 tiles plus one. Step three we have 3 sets of 3 tiles plus one. Step four we have 4 sets of 4 tiles plus one.

So nth term would be n^2+1.

Argument #5

Problem #2: Growing S's

Question 1. Consider the pattern below. How many square tiles would there be in the eighth step of this pattern?



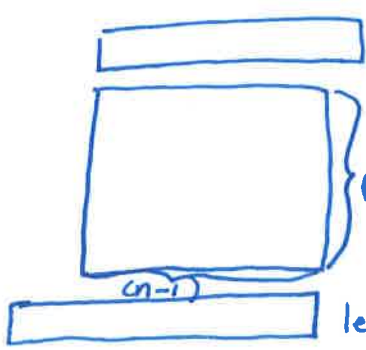
The eigth step of this pattern will have 65 square tiles.

Question 2. Write an expression for the total number of square tiles in the figure at an arbitrary step (n) of the pattern.

Step n : $n + n + (n-1)(n-1)$

$= 2n + (n-1)^2$

Question 3. Prove that your expression is a valid representation of the number of tiles at step n of the pattern. You may use drawings, words, numbers, and/or symbols for your proof.



length of n $\rightarrow$ n

$+$

$\rightarrow$ $(n-1)(n-1)$

$+$

length of n $\rightarrow$ n

$\therefore$ $n + n + (n-1)(n-1)$

$= 2n + (n-1)^2$

Let n be the number of step in this pattern. The first and last rows consist of n number of square tiles. The middle portion consists of the same number of columns and rows of square tiles and is equal to one less than the step number.