# Developing community norms for proof:
# Forum discussions on the nature and import of proof

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*Syllabus language & rubric:*

Six forum assignments will be made throughout the semester, at the end of each week except for the last week. For each forum assignment, I will create a forum in Moodle and assign a prompt by Friday morning to help you reflect on what you are learning about proof and mathematics. Approximately 1/3 of the class will be assigned to post in forum during a given week; each student will therefore post for two of the six forum assignments. The posts should thoughtfully respond to the prompt from personal experience and, as appropriate, raise questions or ideas for further discussion and should be completed before Monday's class. Students who are not assigned to post will participate in a class discussion based on the posts by responding to the posts and to each other. At least two comments are expected each week and should be posted before the Friday after the posts were made, at which time a new prompt will be given. The goal is that we will create a class discussion on each topic so we can all learn from each other's reflections. Which students are responsible for which week's forum posts will be posted with the first forum (with reminders in later weeks).

The following rubric will be used to assess the forum posts and comments, respectively.

| Forum Rubric | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|
| Post | Post follows instructions and shows thoughtful consideration of the assigned topic in a way that prompts discussion. | Post follows instructions and shows thoughtful consideration of the assigned topic. | Post slightly misses the mark, but nonetheless shows evidence of thought and effort. | Post shows little evidence that the author was trying to follow instructions or think carefully about the entry. | Post is absent or completely irrelevant to the assignment. |
| Comments | 4, plus additional thoughtful comments or comments which spur discussion . | Two thoughtful comments are posted. | Two comments are given, but they are perfunctory OR One thoughtful comment is given. | One perfunctory comment OR two irrelevant comments. | No comments given. |

A grade will be given for each week and the scores summed. The sum will be converted to a grade as follows:

27+ = A, 24 = B, 20 = C, 12 = D, 9=F Plusses and minuses will be assigned appropriately.

*Prompts from Fall 2012:*

Week 1: Reflect on proofs

For this forum assignment, I want you to reflect on the nature and importance of proofs in the discipline of mathematics. I strongly encourage you to consider your own experience with proof throughout your life as well as some of the examples we have seen in class and your own writing of the domino and chessboard problem. Here are some questions to guide your reflection, though you do not need to answer all of them:

1. What is the purpose of a proof?
2. One definition of a proof is "a convincing communication that answers why" (Henderson). What do you think this means? Do you agree or disagree with this definition?
3. What do you think are the elements of a good proof, and why are these important?
4. What are the potential difficulties you foresee yourself having to overcome in the process of learning to write good proofs?

Week 2:

Last week in the forum you discussed what a proof was and what you think is difficult about proofs. Many of you reflected on Henderson's definition and what it meant to "communicate" or "convince" someone. This week can be considered a continuation of the discussion - but I am giving you concrete examples to reflect on.

Consider the following (true) statement:

If $n$ is a positive integer, $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

Below are two different proofs of this result - both accepted by the mathematical community as proofs.

(1) A proof without words. A proof without words is a visual way to prove a theorem, without using words. Usually this is a diagram and occasionally an animation. Proofs without words are well-enough accepted by mathematicians that there are books of them and a regular column in the College Mathematics Journal. The first two diagrams of www.cut-the-knot.org/ctk/pww.shtml and the animation at the top of the middle column at www.usamts.org/About/U_Gallery.php both illustrate a proof of this result, although in slightly different ways that you may find more or less convincing.

(2) An algebraic proof:

Consider $S = 1 + 2 + 3 + \cdots + n$. We want to find the sum, $S$. We add:

$$S = 1 + 2 + 3 + \cdots + (n - 1) + n$$

$$S = n + (n - 1) + (n - 2) + \cdots + 2 + 1$$

to get $2S = (n + 1) + (n + 1) + (n + 1) + \cdots + (n + 1) + (n + 1)$, a sum with $n$ terms. Therefore, $2S = n(n + 1)$ and $S = \frac{n(n+1)}{2}$.

After trying to understand these proofs, reflect on them in light of what you think a proof is and Henderson's definition. You might answer some of the following questions, or add your own reflections on proof in light of these examples:

- Which proof do you find most convincing? (There are two proofs without words and the algebraic proof above.) Or are none convincing? What makes a proof convincing or one proof more convincing than another? Does something convince you that might not convince someone else?
- How are these proofs communicated? Does that affect whether you find it convincing? Is there an assumption about who the audience is that affects the communication?
- Henderson also talks about a proof answering "why?" Do you find any of these proofs answer that question better than the others?
- Are there ways in which the categories of convincing, communicating and answering why don't explain why you find one proof better than another? Consider some of the things you discussed in the forum - logical reasoning, validity at each point, showing something true or false, etc.
- Would words make the proof without words more convincing to you? What words would you add?
- Are there areas you struggle to understand these proofs?

Week 3: Proof & Truth Part I

Over this week and next week, you will read the chapter "Proof and Truth" from the book *Mathematics Through the Eyes of Faith.* For this week, read p. 115-p.131 at the section break. The goal of the forum this week is to discuss the reading– any topics related to the reading are fine. Here are some suggestions if you lack other ideas:

- How does this reading compare to our discussions so far about the nature of proof? You might consider specific examples used or the attempt to define proof.
- What does it mean to prove something impossible and how does this differ from the way we tend to use the word "impossible" in daily conversation?
- What are the benefits and downfalls of axiomatic systems? What do you think the implications of Gödel's theorems about axiom systems are?
- What is your reaction to the fact that mathematicians don't always agree on what a proof is?
- How does the idea of proof relate to your understanding of truth (or Truth)?

Week 4: Proof & Truth Part 2

Last week, you began discussing the relationship between proof and truth based on the chapter "Proof and Truth" from the book *Mathematics Through the Eyes of Faith.* This week you should complete this chapter (p.131-p.138) and we will continue this discussion with a focus on the relationship between proof and Christian truth.

While any discussion regarding the relationship between proof and truth is acceptable, here are some specific questions that you might consider in your discussion:

- How are mathematical truth and Christian truth similar or different?
- How does the role of context affect reasoning in mathematical proof or your Christian faith? Are there similarities or differences?
- What is the role of trust or faith in mathematics? In Christian truth? How are these the same or different?
- Does our belief in God justify our study of mathematics? Why or why not?

Week 5: Math Talks

Over the last few weeks, you were supposed to attend (or if you couldn't, watch) a math talk. This forum is intended to help you think about and discuss proof in the context of a math talk and how it compares to written proof as we have been discussing it in class. Here are some questions that might help get you started in your reflections – you may answer as few or as many questions as you wish.

- How was proof used in the talk you saw? How does this compare to what you are hearing from your classmates regarding different talks they viewed?
- What other elements of the talk were present besides proof? Did the nature of the content affect the number or type of proofs or justification used?
- Were the proofs in the talk completely explained and rigorous? Explain the ways in which you thought they were/were not rigorous.
- Based on your experience and your own ideas, does communicating in speech rather than writing affect the elements of a proof? This might include adding and subtracting, or changing the style or relative importance of elements. (Yes, you can go back to "a convincing communication that answers why" if you wish!)

Week 6: Proof & Beauty

*Whatever is true, whatever is noble, whatever is right, whatever is pure, whatever is lovely, whatever is admirable—if anything is excellent or praiseworthy—think about such things. Philippians 4:8*

We have been discussing what is true and how that relates to proof. In this forum, you will take our discussions on proof in a different but related direction and consider the aesthetics of proof – what makes it lovely? Here are some possible questions for discussion:

- Can a proof be beautiful? Most mathematicians think so – but do you agree? Is this beauty the same as artistic beauty or qualitatively different? Why or why not?
- What makes a proof beautiful? Or at least aesthetically pleasing? Does the way it is written or presented make a difference?
- In discussing beauty in mathematics, some mathematicians have identified characteristics like symmetry, surprise or paradox, elegance, brevity/simplicity, originality, and power. What do these characteristics mean? Do you think these characteristics can make mathematics beautiful? If so, what examples can you think of? If not, why not?

*Original posts:*

To address this question, we need some common examples to talk about! Original posters should each find a proof they think is aesthetically pleasing in some way (possible sources below), make it available for others to read (either scan it in and attach or give the link to it), and then explain what they find aesthetically pleasing about the proof. Try to identify some general characteristics and how you see them specifically in this proof. Remember, original posters should provide a significant reflection and can pose questions for others to consider.

*Comments:*

Commenters should read the proofs that the original posters scanned in and then respond to the discussion about characteristics of beautiful mathematics, keeping in mind the guiding questions above. Remember to post at least 2 thoughtful comments.

*Possible sources:*

There is no restriction on where you find an aesthetically pleasing proof – only that it be a proof. Here are some places to look for a beautiful proof if you don't know one already!

In the library on reserve for this week:

- *Charming proofs: a journey into elegant mathematics* by Claudi Alsina - a book specifically about proofs that others find elegant.

- *Proofs from the Book* by Martin Aigner & Gunther Ziegler– as noted in your last reading, the late mathematician Paul Erdos spoke of a book that God kept of the most beautiful proofs of each true statement. These are proofs that some mathematicians think are in God's book. (2nd and 4th editions on reserve)

In math student space on the counter:

- A bunch of books by Ross Honsberger –*Mathematical Morsels, Mathematical Gems, Mathematical Plums, etc.*
- *Aha! Solutions* by Martin Erickson – another set of problems and solutions.