TEST 2

Probability

INTRO TO PROBABILITY NOTES

PROBABILITY is a branch of mathematics that studies the likelihood that a certain event will occur. Probability is a continuation of combinatorics, so we will use the same terminology.

PROBABILITY OF EVENT E: $P(E) = \frac{n(E)}{n(S)} = \frac{\#\ of\ elements\ in\ E}{total\ \#\ of\ possible\ outcomes}$

Ex1. Roll one die P(rolling an even) = P(rolling # $\geq$ 2) =

Ex2. Toss a coin twice

P(one H and one T) =

Ex3. Toss a coin three times

P(exactly 2 H's) = P(2 or 3 H's) = P(exactly 1 T) =

Properties of Probabilities: 1) $0 \leq P(E) \leq 1$ all probabilities must be between 0.0 and 1.0 (0% and 100%)

2) if $P(E) = 1$ then E always happens (100% of the time)

Ex. P(rolling a 6 or less)

3) if $P(E) = 0$then E never happens (0% of the time)

Ex. P(rolling 7)

Ex4. Draw one card from a deck

P(♦) = P(jack) = P(3♠) = P(face card) =

ODDS in favor of event E **a to b** (or **a:b**)means every **a** times event E occurs, **b** times event E does not occur

Example: if odds are 2:3 that Street Sense will win Kentucky Derby again,
then every 2 times he wins, 3 times he'd lose

Odds in favor of E are **a:b**, then $P(E) = \frac{a}{a+b}$ (Note: odds against E are b:a)

Ex1

Probability of E occurring	Odds in favor of E occurring	Odds against E occurring
P(E) = $\frac{a}{a+b}$	a to b	b to a
$\frac{5}{17}$		
	3 to 8	
		20 to 11
0.9		

Ex2. A fair die is rolled, what are the odds in favor of rolling an even #?

Ex3. In a contest, the odds against winning 1st prize are 100 to 1, what is the probability of winning?

MUTUALLY EXCLUSIVE: events that can't occur at the same time $A \cap B = \emptyset$ which means that:

$$\boldsymbol{P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)}$$

Ex1. P(rolling a 3 or a 4)

Ex2. P(draw a ♦ or a black card)

ADDITION RULE FOR PROBABILITIES for any event A and B, $\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

Ex1. Given $P(A) = 0.6, P(B) = 0.5$, and $P(A \cap B) = 0.2$; then $P(A \cup B) =$

Ex2. P(drawing a 3 or a ♥) =

Ex3. A bag contains 2 blue, 3 red, and 4 white poker chips and two are chosen at random at the same time.

a) P(both red)

b) P(1 blue and 1 white)

c) P(neither red)

d) P(both white or both blue)

e) P(at least one red)

COMPLEMENT E^C for event E, $\boldsymbol{P(E^C) = 1 - P(E)}$

The idea is that the probability of an event happening plus the probability of the event not happening equals one.

NOTE: P(at least one) = 1 – P(none)

Ex. You flip a coin 4 times.

a) P(at least 1 tail)

b) P(at least 3 tails)

Homework Problems:

1) A gym lock has a disk with 36 numbers written around its edge. The combination to the lock is made up of three numbers (such as 12-33-07) with repetition allowed.

a) How many different possible combinations are there for this lock?

b) What is the probability of randomly guessing the correct one?

2) What is the probability of seeing exactly 3 heads when a coin is tossed five times?

3) A game at a carnival has 75 rubber ducks floating in water. The ducks are numbered 1 to 75 written on their undersides. To play you select a duck and look at the number. If the number is less than 60, you win a piece of gum. If the number is at least 60 but less than or equal to 70, you win a small stuffed animal duck. If it's greater than 70, you win a giant stuffed animal duck. What is the probability that you:

a) win a small stuffed duck?

b) don't win a duck (small or giant)?

4) A letter is chosen randomly from the alphabet. What's the probability that letter is in the word CRIBBAGE or YAHTZEE?

5) A bookmaker has placed 8 to 3 odds against a particular football team winning its next game. What is the probability, in the bookmaker's view, of the team winning?

6) In March 2004, vegas.com gave odds on who would win the 2004 World Series. They gave the New York Yankees 2:1 odds and the Boston Red Sox 5:2 odds.

a) Find the probabilities that each of the teams will win according to these odds.

b) Who did they think was more likely to win the World Series?

7) In horse racing, the betting odds are based on the probability that the horse does not win.

a) In a past Kentucky Derby, the betting odds for the favorite horse, Point Given, were 9 to 5. Compute the probability that Point Given would win the race.

b) In the same race, the betting odds for the horse Monarchos were 6 to 1. Estimate the probability that Monarchos would *lose* the race.

c) Invisible Ink was a long shot, with betting odds of 30 to 1. Use these odds to estimate the probability that Invisible would win the race.

8) The game board for the television show Jeopardy is divided into six categories with each category containing 5 answers. In the Double Jeopardy round, there are two hidden Daily Double squares.

a) What is the probability that a contestant chooses a Daily Double square on the first turn?

b) What are the odds in favor of choosing a Daily Double square on the first turn?

9) From a deck of cards, you turn over a 5 and then a 6. What is the probability the next card you turnover will be a face card?

10) The lottery in an extremely small state consists of picking two different numbers from 1 to 10. Ten numbered ping pong balls are dropped in a fish bowl and two are selected. Suppose you bet on 2 and 9. What is the probability that

a) you match both numbers?
b) you match at least one number?

11) Two letters are chosen at random from the word CROQUET. What is the probability that both are vowels?

12) Suppose you are dealt three cards from a regular deck of cards. What is the probability that all three will be jacks?

13) In a Win, Lose, or Draw class, five of the twelve females are blonde and six of the fifteen males are blondes. What is the probability of selecting a female or a blonde?

14) You have played soccer for many seasons and have stored all of the leftover socks in a drawer. You've lost a few, so now you have 4 red, 5 green, and 7 blue socks. If you reach in and grab 2 socks at random, what is the probability:

a. both are red?
b. both are green?
c. both are blue?

d. neither is red?
e. at least one is red?
f. the socks don't match?

15) You roll a fair die four times. What is the probability that you roll at least one 6?

16) You and a friend each pick a card from a regular deck of cards but do not look at them. Both of you then hold your cards up by your foreheads so that you can see each other's card but you can't see your own. Your friend is holding a 6. What is the probability that your card is higher (assume Aces are high)?

17) To win the jackpot of a state lottery game, you must correctly pick the six numbers selected from the given numbers (order doesn't matter). What is the probability of winning the state lottery in the following states:

a) Massachusetts #'s 1 through 36

b) New York #'s 1 through 40

18) Two cards are drawn at the same time from a standard deck. What's the probability of getting:

a) both aces or both face cards?

b) both black or both face cards?

c) both aces or both black?

19) There are 15 females and 10 males in a class. A group of 6 is to be formed to make a presentation at a conference. Find the following.

a) P(all females)

b) P(all males)

c) P(2 females)

d) P(3 males)

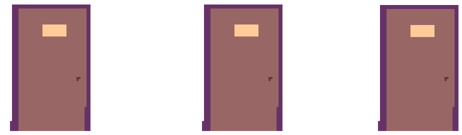
e) P(at least five females)

f) P(at least one male)

20) What is the probability of getting at least 5 heads if you flip a coin 7 times in a row?

Group #15

The Monty Hall Problem
Let's Make a Deal



Door #1 Door #2 Door #3

There used to be a game show hosted by Monty Hall called Let's Make a Deal which started back in 1963 in which a contestant would get whatever was behind either Door #1, #2, or #3. Two of the doors had a goat (or nothing) behind them and the other door had a car (or some other nice prize). The contestant was asked to choose one of the doors. Once the contestant had chosen, at least one of the remaining doors had a goat behind it. Monty would then open a remaining door with the goat behind it and offer the contestant a chance to switch doors with the other remaining door.

The Monty Hall Problem: **Is it better for the contestant to stay or switch?**

In order to answer this question, we will have each group do a series of trials with members acting as contestants. Staying and switching on subsequent turns. Then we will compile all of our data and see what conjectures we can make.

15.1) Before we do any trials what do you think: is it better for the contestant to stay, switch, or it doesn't make a difference?

15.2) Take 3 cards from your deck- one Ace (acting as the car) and two jacks (acting as the goats). Each member of your group will act as the contestant ten times.

a) Have someone other than the contestant place the three cards face down in a row without the contestant seeing.

b) Have the contestant choose one of the cards.

c) Of the cards the contestant didn't choose, flip over a remaining jack and ask if the contestant wants to stay or switch.

d) First 5 times for each contestant, **stay**. Record if they won (got ace) or lost (got a jack) when they stayed.

e) Last 5 times for each contestant, **switch**. Record if they won (got ace) or lost (got a jack) when they switched.

	WON		LOST	
STAY				
SWITCH				

15.3) After seeing your group totals, do you think you should stay or switch?

15.4) Now we'll look at the actual probabilities to get a theoretical answer. If you choose correctly with your first choice, it's better to stay. What's the probability that you choose correctly right off the bat?

15.5) If you chose incorrectly with your first choice, it's better to switch. What is the probability that you don't choose the right door for your first choice?

15.6) So what does it look like would be better to do, stay and switch? Explain.

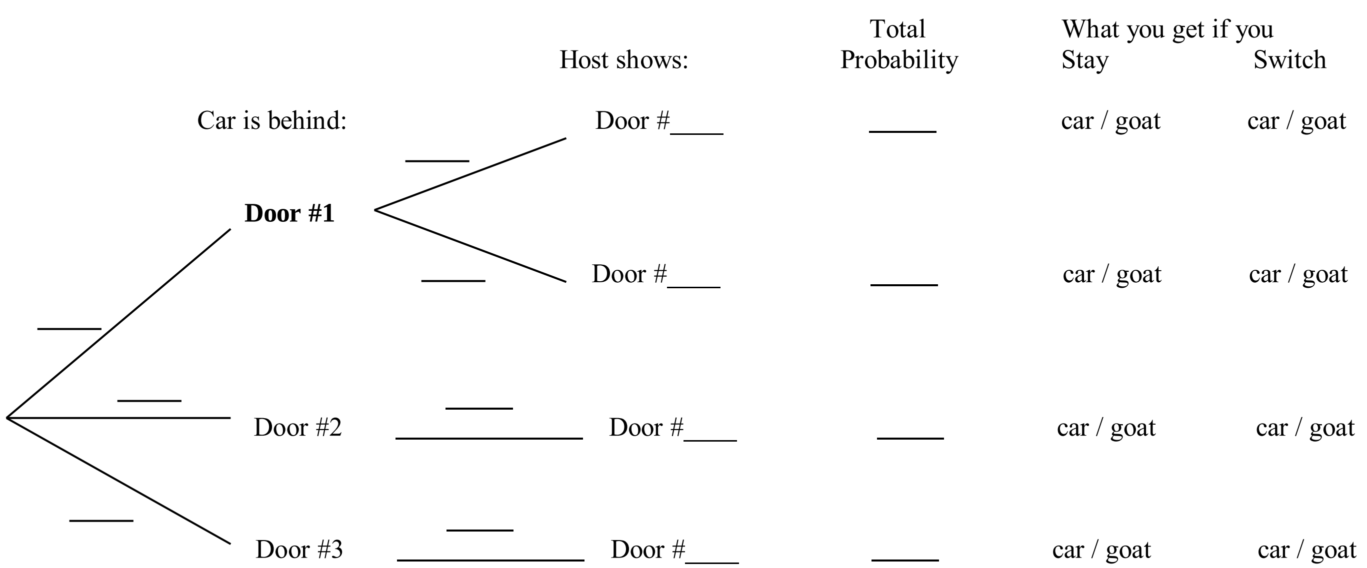
15.7) We'll draw a tree now (called a decision tree, each bud produces branches whose probabilities must add up to 1) to better illustrate this. Let's assume the **contestant chooses Door #1**.

1) Add a probability to each tree branch.

2) Add the door number for each door the host could show in each situation.

3) Fill in the total probability by multiplying the branches.

4) Say what you would get if you either stay or switch from your first pick of Door #1.

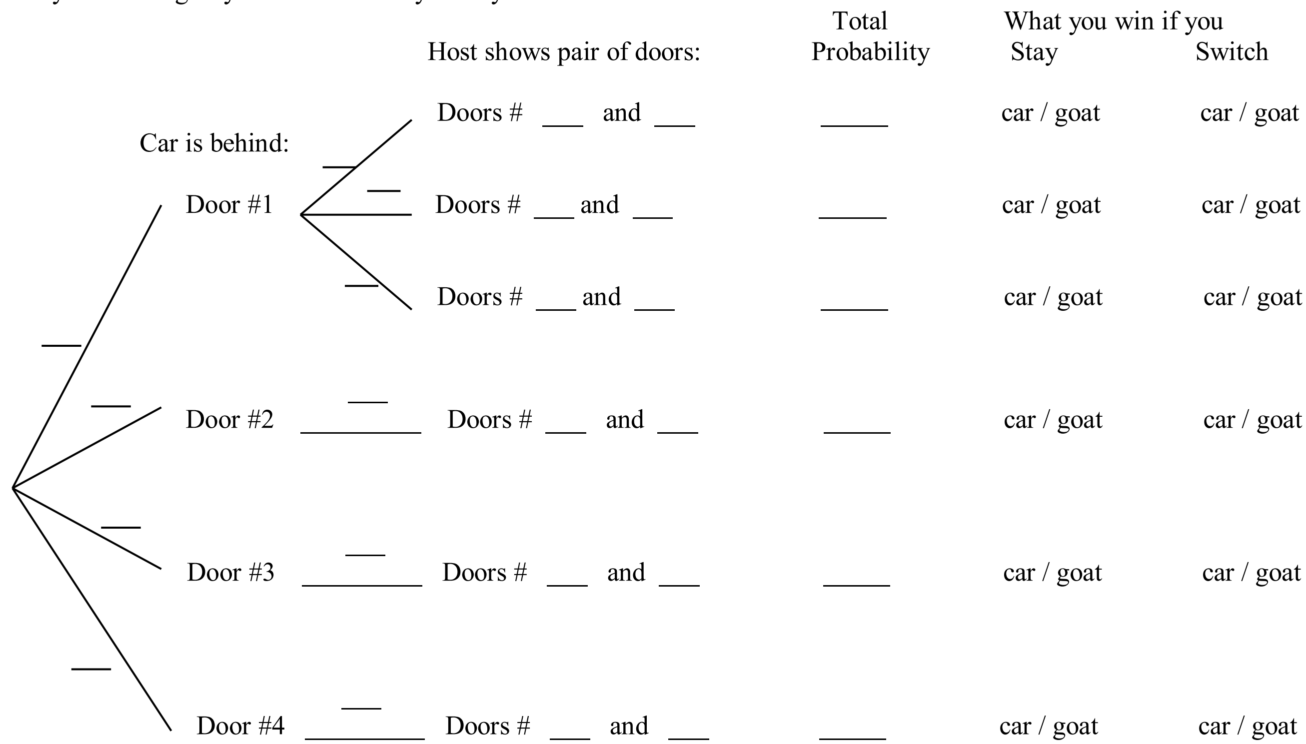


15.8) From this tree, what is the probability of getting

a) a goat if you switch? b) a car if you stay? c) a goat if you stay? d) a car if you switch?

15.9) Does this support what you found in your group and in 6)? Explain.

15.10) What changes if there are four doors (one with a car and 3 with a goat), and you get to see what is behind two of them after you make your initial choice of door? Draw a tree as in 7) assuming **the contestant chooses Door #1** and determine the probability of winning of you switch and if you stay.



15.11) From your tree above determine the following:

a) P(car if you stay) b) P(car if you switch) c) Stay or switch?

15.12) a) What if there were 5 doors and you get to see what's behind 3 of them?

i) P(car if you stay) ii) P(car if you switch)

b) What if there were 10 doors and you get to see what's behind 8 of them?

i) P(goat if you stay) ii) P(goat if you switch)

c) What if there are N doors and you get to see what's behind N – 2 of them after you make your choice of doors?

i) P(car if you stay) ii) P(car if you switch)

15.13) Another variation… There are four doors (one with a car and 3 with a goat). You pick a door and then the host will open one of the goats and give you the option of switching. The host will then open a second goat and you again decide to stay or switch. What is the probability of winning the car if you stick with your original choice? What do you suppose is your best strategy, stay with original, switch with first offer, or switch with second offer? Discuss with your group.

Odds &Probability Questions

Convert the given probabilities into odds in favor of E happening.

15.14) a) P(E) $= \frac{4}{5}$ b) P(E) $= \frac{1}{8}$ c) P(E) $= 0.28$

Odds = _______ Odds = _______ Odds = _______

Convert the given probabilities into odds of E NOT happening.

15.15) a) P(E) $= \frac{3}{17}$ b) P(E) $= \frac{12}{40}$ c) P(E) $= 0.8$

Odds against = _______ Odds against = _______ Odds against= _______

Convert the given odds in favor of E occurring to probabilities (leave in fraction form).

15.16) a) 5:1 b) 3:2 c) 9:5

P(E) = _______ P(E) = _______ P(E) = _______

Convert the given odds against E occurring to probabilities (leave in fraction form).

15.17) a) 4:5 b) 9:1 c) 2:9

P(E) = _______ P(E) = _______ P(E) = _______

15.18) If a card is drawn at random from a standard deck of cards, what are the odds against the card being an ace?

15.19) A survey conducted by the U.S. Bureau of the Census in 1997 showed that 84% of the people in the United States are covered by health insurance. What are the odds a person in the United States is covered by health insurance?

15.20) The odds against winning a $2 prize in the Lucky Gold game from the Colorado Lottery are 9:2. What is the probability of winning $2 in the game?

15.21) If a game spinner has a 3/8 probability of landing on blue, what are the odds of the spinner landing on blue?

15.22) In the game of Monopoly, players roll two dice and move the number of spaces as indicated by the sum of the two dice. Pennsylvania Avenue is five spaces from Boardwalk on the game board. What are the odds that a player on Pennsylvania Avenue will land on Boardwalk by rolling the dice? Hint: there are 36 different possible outcomes when rolling two dice.

Group #16

Dice Games:
Risk and Craps

There are many, many games which require you to roll dice in order to play. In this group we will focus on Risk and Craps. First we will look at rolling a pair of dice and how many different outcomes there could be.

With perfectly balanced dice, each combination on a pair of dice is equally likely. The table below shows all of the possible outcomes for rolling a pair of dice (upper left number is die #1 and lower right number is die #2). You can think of them as 2 different colors so that you can tell them apart.

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

16.1) How many total outcomes are there?

16.2) From the above table, knowing all outcomes are equally likely, determine the following:

a) P(doubles)

b) P(sum of 2)

c) P(sum of 3)

d) P(sum of 4)

e) P(sum of 5)

f) P(sum of 6)

g) P(sum of 7)

h) P(sum of 8)

i) P(sum of 9)

j) P(sum of 10)

k) P(sum of 11)

l) P(sum of 12)

m) P(sum of 3 or 10)

n) P(sum of at least 9)

o) P(sum of at most 5)

RISK In the game of Risk, the board is just a map of the world with continents sectioned off. You and your opponent place armies on different territories and then attack each other to try to attain more territories (the goal is world domination). When an attack is made, **the attacker gets to roll three dice and the defender only rolls two**. The attacker's highest die number is compared to the defender's highest die. The attacker's next highest die is compared against the defender's second highest die. The attacker's lowest die does not count. **The defender wins in the case of a tie**. With each dice comparison, the loser removes one army from his territory on the game board. So there are three things that can happen in an attack:

1) the attacker loses 2 armies
2) the defender loses 2 armies
3) they split and each loses one army

Example of each: 1) Attacker rolls 6, 3, 3 and defender rolls 6, 4, then the attacker will lose two armies.
2) Attacker rolls 4, 5, 2 and defenders rolls 3, 4, then the defender will lose two armies.
3) Attacker rolls 1, 2, 5 and defender rolls 5, 1, then they both lose one army.

PLAY MINI GAME OF RISK With the given dice, play this mini game of Risk with your group. Each side of the table starts with 10 pennies (armies). Take turns being the attacker and defender and lose your pennies according to the rules above.

For the next problems, a scenario is given with one or two of the dice rolls unknown. You are to determine probabilities of each type of result. Here are a couple of examples:

Ex1. Attacker: 5, 3, __ Defender: 5, 4

a) P(attacker loses 2 armies) b) P(defender loses 2 armies) c) P(both lose one army)

__ = 4, 3, 2, 1 prob 4/6 __ = 6 prob 1/6 __ = 5 prob 1/6

Ex2. Attacker: 1, 6, 4 Defender: __, __

a) P(attacker loses 2 armies)
__/__ = 6/6, 6/5 (5/6), 6/4 (4/6)

Prob: 5/36

b) P(defender loses 2 armies)
__/__ = 5/3 (3/5), 5/2(2/5), 5/1(1/5), 4/3 (3/4), 4/2(2/4), 4/1 (1/4), 3/3, 3/2 (2/3), 3/1 (1/3), 2/2, 2/1 (1/2), 1/1

Prob: 21/36

c) P(both lose one army)
__/__ = 6/3 (3/6), 6/2 (2/6), 6/1 (1/6), 5/5, 5/4 (4/5), 4/4

Prob: 10/36

Hint: Use your grid on the previous page to help when you're looking for two unknown dice.

For the following problems, determine the probabilities by first listing the outcomes for each category.

16.3) Attacker: 3, 6, 2 Defender: 4, __

a) P(attacker loses 2 armies) b) P(defender loses 2 armies)

c) P(both lose one army)

16.4) Attacker: 6, 2, __ Defender: 2, 3

a) P(attacker loses 2 armies) b) P(defender loses 2 armies)

c) P(both lose one army)

16.5) Attacker: 3, 1, __ Defender: 1, 3

a) P(attacker loses 2 armies)

b) P(defender loses 2 armies)

c) P(both lose one army)

16.6) Attacker: 4, 1, 6 Defender: 2, __

a) P(attacker loses 2 armies)

b) P(defender loses 2 armies)

c) P(both lose one army)

16.7) Attacker: 3, 2, __ Defender: 4, 2

a) P(attacker loses 2 armies)

b) P(defender loses 2 armies)

c) P(both lose one army)

16.8) Attacker: 3, 3, 3 Defender: 3, __

a) P(attacker loses 2 armies)

b) P(defender loses 2 armies)

c) P(both lose one army)

16.9) Attacker: 3, 5, 4 Defender: __, __

a) P(attacker loses 2 armies)

b) P(defender loses 2 armies)

c) P(both lose one army)

RULES FOR THE GAME OF CRAPS

1. Roll a pair of dice.

*Sum of 7 or 11 (**natural**), you win an even money payout (for a $1 bet, you'll get $1 plus keep your bet)*
*Sum of 2, 3, or 12 (**craps**), you lose your bet*
*Sum of 4, 5, 6, 8, 9, or 10 is called your **point**.*

2. If you roll a point on your first roll, then you continue to roll the dice.

If you roll your point again before rolling a 7 then you win an even money payout.
If you roll a 7 before you roll your point again, you lose your bet.

Have each person at your table be the "shooter" for two rounds of craps.

16.10) a) What is the probability that you roll a natural on your first roll? i.e. P(sum of 7 or 11)

b) What is the probability that you "crap out" on your first roll? i.e. P(sum of 2, 3, or 12)

c) What is the probability that you get a point on your first roll? i.e. P(sum of 4, 5, 6, 8, 9, or 10)

d) After you roll a point on your 1st roll, what is the probability you roll a sum 7 on the 2nd roll and lose?

e) After you roll a point on your first roll, what is the probability that you don't roll a 7 on the 2nd roll?

f) After you roll a point on your 1st roll and neither a sum of 7 or your point on the second roll, what is the probability that you roll a 7 on your 3rd roll? Did this probability change at all from d)? Why or why not?

16.11) If two dice are rolled, what is the probability that:

a) you don't roll doubles?

b) the sum is odd?

c) you roll a 1 on one or both of the dice?

d) the sum is 7 or 8?

e) the sum is not 7 nor 8?

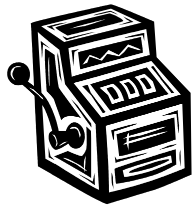
f) the sum is at least 7?

g) the sum is at most 6?

h) the sum is 14?

16.12) What sum has the highest probability of being thrown when a pair of dice is rolled? What is its probability?

Group #17



One Arm Bandits (Slot Machines)

Questions are taken from
The Slot Machine Answer Book 2nd ed. 2005 by John Grochowski
See how many of the following you can get right!

1) In the US, slot machines account for:
   a) about 40% of casino revenue
   b) about 50% of casino revenue
   c) about 60% of casino revenue
   d) more than 70% of casino revenue

2) The number of slot machines in use in the US (as of 2011) is:
   a) more than 200,000
   b) more than 400,000
   c) more than 600,000
   d) more than 800,000

3) The most common coins played in US slot machines are worth:
   a) 5 cents
   b) 25 cents
   c) 50 cents
   d) one dollar

4) Outside Nevada, the state with the most slot machines is:
   a) New Jersey
   b) Mississippi
   c) Illinois
   d) Missouri

5) The largest slot jackpot ever hit in the US is:
   a) $9 million
   b) $19 million
   c) $29 million
   d) $39 million

6) Playing more than one machine at a time:
   a) narrows the house edge.
   b) ensures the player of finding at least one loose machine.
   c) costs the player more money in the long run.

7) Instead of cash, early slot machines sometimes paid:
   a) golf balls
   b) chewing gum
   c) cigars
   d) all of the above

8) Orange, plum, and lemon symbols, all still used on slot machines today, were introduced on slot machine to signify:
   a) the inventor's roots as a fruit-stand vendor.
   b) flavors of chewing gum.
   c) wealth, as fresh fruit then was so expensive.
   d) none of the above.

9) The device with inner working most like a modern slot machine is:
   a) a 1960s slot machine
   b) a coin-operated gumball machine
   c) an electric clock
   d) a personal computer

10) The odds of winning on an old-style mechanical slot machine are determined by:
   a) the number of symbols and spaces on each reel.
   b) the number of reels.
   c) both of the above.
   d) none of the above.

11) The odds of winning on an modern mechanical slot machine are determined by:
   a) the number of symbols and spaces on each reel.
   b) the number of reels.
   c) both of the above.
   d) none of the above.

12) A machine is programmed to pay a jackpot about once per 10,000 pulls. You have played 9,999 pulls without hitting a jackpot. The odds of hitting on the next pull are:
   a) even
   b) 1 in 100
   c) 1 in 10,000
   d) undetermined

13) A machine is programmed to pay a jackpot about once per 10,000 pulls. You just won a jackpot on your last pull. The odds of hitting again on the next pull are:
   a) 1 in 10,000
   b) 1 in 100,000
   c) 1 in 1 million
   d) undetermined

14) After playing a slot machine for 2 hours, you walk away. The player who replaces you immediately hits the jackpot. If you had stayed:
   a) You would have hit the jackpot.
   b) You would have had about a 50% chance of hitting the jackpot.
   c) You chances of hitting the jackpot would have been very small.

15) Slot players increase their chances of winning by playing:
   a) Slots on which they have seen others winning.
   b) Slots on which they have seen others losing.
   c) Slots that haven't been played in a few hours.
   d) none of the above.

17.1) A slot machine has 4 reels and each reel has 8 different symbols (each including one jackpot).

a) How many different results could occur in one spin?

b) What is the probability of getting all 4 jackpots?

c) What is the probability of getting no jackpots?

d) What is the probability of getting 1 jackpot?

e) What is the probability of getting 2 jackpots?

17.2) Another slot machine has 8 reels each with 4 different symbols, each including one jackpot.

a) How many different results could occur in one spin?

b) P(8 jackpots)

c) P(no jackpots)

17.3) A slot machine has 3 reels each with 10 symbols. There is one jackpot symbol on each reel.

a) How many different results could occur in one spin?

b) What is the probability of spinning:

i) all three jackpots?

ii) jackpot any other symbol jackpot?

iii) exactly 2 jackpots?

iv) at least 2 jackpots?

17.4) A slot machine named Big Blue has 4 reels each with 11 different symbols. There is one bulldog on each reel.

a) How many different results could occur in one spin?

b) P(no bulldogs)

c) P(3 bulldogs)

d) P(at least 1 bulldog)

e) P(at least 3 bulldogs)

17.5) A slot machine has 5 reels each with 7 different symbols (including one jackpot on each reel).

a) How many different results could occur in one spin?

b) P(no jackpots)

c) P(1 jackpot)

d) P(2 jackpots)

e) P(3 jackpots)

f) P(4 jackpots)

g) P(5 jackpots)

17.6) Jokers Wild is a slot machine that has 3 reels each with 20 symbols. Its first reel has 4 joker symbols; second has 3 joker symbols; and third has 2 joker symbols.

a) How many different results could occur in one spin?

b) P(3 jokers)

c) P(no jokers)

d) P(at least 1 joker)

e) P(2 jokers)

17.7) A slot machine has 4 reels each with 15 symbols. The first reel has 3 jackpots; the second also has 3 jackpots, the third has 5 jackpots; and the fourth reel has 2 jackpots.

a) How many different results could occur in one spin?

b) P(4 jackpots)

c) P(no jackpots)

d) P(1 jackpot)

MORE PROBABILITY NOTES

CONDITIONAL PROBABILITY

P(rolling a 3) = , but what if we know that we rolled an odd number?

P(rolling a 3 | rolled odd) =

P(rolling a 3 | rolled even) =

$$\boldsymbol{P(B \mid A) = \frac{P(A \cap B)}{P(A)}}$$

"the probability of B given A"

(given that event A occurred, we determine the probability of B occurring)

Ex1. Using the formula, find P(rolling a 3 | rolled odd) =

Ex2. A card is drawn from a deck.

a) P(spade | not red)

b) P(red | not a spade)

c) P(face | spade or jack)

Ex3. Out of 100 students, 50 play soccer, 70 play basketball, and 40 play both. Find the probability that a student chosen at random plays:

a) neither sport.

b) basketball but not soccer.

c) soccer given they play basketball.

d) basketball given they don't play soccer.

e) doesn't play soccer given they don't play basketball.

INDEPENDENT 2 events A and B are independent if A doesn't affect B

$$\boldsymbol{P(B \mid A) = P(B)} \qquad \text{or} \qquad \boldsymbol{P(A \cap B) = P(A) \cdot P(B)}$$

Ex1. One card is drawn from a deck, replaced, and then a second card is drawn. Find the probability that

a) both are face cards.

b) 1st card is an ace and 2nd is not an ace.

Now let's say a card is drawn from a deck, NOT replaced, and then a 2nd is drawn. Answer the same as above:

a)

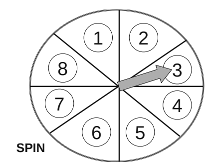
b)

Ex2. At a wrestling match you buy a bag of M&M's. The bag contains the following colors: 12 red, 12 blue, 7 green, 13 brown, 3 orange, and 10 yellow, you grab 3 candies from the 57 M&M's in the bag.

a) Find the probability of getting 2 browns and 1 orange.

b) Find the probability that 1st is brown, 2nd is orange, and 3rd is another brown if you grab the candies one at a time without replacement.

HOMEWORK PROBLEMS:



1) If you were to spin the wheel, it is equally likely to stop at any point. Find the probability of spinning

a) a 5 b) an even number c) at least a 3

d) at most a 3 e) any number but 6 f) a 7 on 1st spin and an 8 on 2nd spin

g) 4 given that you spun an even h) a 5 or 6 given that you spun at least a 4

i) 4 given that you spun an odd j) odd given that you spun at most a 5

2) Someone flips 3 coins (a penny, a nickel, and a dime) behind a screen and says,

a) "I flipped two heads." What is the probability that the penny landed on heads?

b) "I flipped at least two heads." What is the probability that they flipped 3 heads?

c) "I didn't flip all tails." What is the probability that they flipped 3 heads?

3) A pair of dice (one green, the other red) is rolled and the sum is noted. Find the probability that:

a) rolled doubles, given that the sum is 8. b) one die is a 6, given the sum is 7.

4) Out of 250 third-grade girls, 120 played volleyball, 140 played soccer, and 50 played both. Find the probability that a girl chosen at random played:

a) neither sport

b) exactly one sport

c) soccer, but not volleyball

d) soccer, given she played volleyball

e) volleyball, given she didn't play soccer

f) didn't play soccer, given she didn't play volleyball

5) A card is drawn from a standard deck. What's the probability that the card is:

a) a queen, given that it's a face card?

b) a 2, given that it's a face card?

c) a face card, given that it's not a 10?

d) an 8, given that it's not a face card?

e) an ace, given that it's neither red nor a face card?

6) There are 25 students in a class with 15 females and 10 males. Three students are needed to represent the class in a competition.

a) What is the probability that 2 females and 1 male are chosen?

b) If the three students are chosen one at a time, what is the probability that the first is a female, the second is a male, and the third is another female?

7) Of 100 athletes, 30 wrestle, 25 play hockey, 40 play softball, 6 wrestle and play hockey, 8 play hockey and softball, 9 play softball and wrestle, and 4 play all three sports.

a) P(wrestle | play softball)

b) P(play both hockey and softball | wrestle)

c) P(don't play softball or wrestle | don't play hockey)

Group #18

Roulette:
What Are the Odds?

Roulette is the oldest casino game still being played. The American roulette wheel has 38 numbered compartments, the numbers 1 through 36, plus 0 and 00 (**which are not considered even**). Players place bets by putting their chips on the appropriate spot on the roulette table (shown). Then the dealer spins the wheel and drops a ball onto the spinning wheel; once the ball starts its descent (starts dropping to the numbers), the dealer calls “No more bets”. When the ball finally stops, that compartment it lands in is the winning number. At this point the dealer places a marker on the winning number, sweeps all of the losing bets off the layout, and then pays the winning bets starting with the “outside” bets (left and side of table on next page) and finishes with the “inside” bets (right side on the numbers). The dealer pays them according to the House Odds which we will compare to the true odds.

18.1) Fill in the table with the probabilities of each bet and then their true odds (usually given as against winning).

What you bet on	**Probability**	**True Odds Against Winning**		**House Odds**
		b to a	**c to 1**	
single # (*plain*)				35 to 1
two #’s (*split*)				17 to 1
three #’s (*street*)				11 to 1
four #’s (*corner*)				8 to 1
five #’s (*5-bet*)				6 to 1
six #’s (*line*)				5 to 1
twelve #’s (*column* or *dozen*)				2 to 1
low 1 to 18(or *high* 19 to 36)				1 to 1
even (or odd)				1 to 1
red (or black)				1 to 1

18.2) Why do you suppose casinos use House Odds instead of True Odds?

18.3) We will now compare the payoffs for certain bets using the actual odds and the house odds. Let's say that you place a **$10 chip** on the following areas of the table (in other words you make a $10 bet).

If you win $10 bet on the following, how much would you win (plus you get to keep your bet on the table)?

a) **black** true odds payout ________

House odds payout ________

b) **1st 12** ture odds payout __________

House odds payout __________

c) **13b,14r, 15b,16r, 17b,18r** true odds payout __________

House odds payout __________

d) **0, 00, 1r, 2b, 3r** true odds payout __________

House odds payout __________

e) **28b, 29b, 31b, 32r** true odds payout __________

House odds payout __________

f) **2b, 5r** true odds payout __________

House odds payout __________

g) **19r, 20b, 21r** true odds payout __________

House odds payout __________

h) **13b** true odds payout __________

House odds payout __________

PLAY ROULETTE Now we'll all play roulette and make bets. Using one of the roulette tables for your group, everyone in the group place a bet where you want to make a bet.

The wheel will be spun and those who win, get the house odds pay out plus they get to keep their bet.
Those who lose, lose their bet.

To place bets on something that is not just one square (a single #, 1 to 18, even, red, 1st 12, etc):

For a 2 # split, place the bet on the line between the two numbers.
For a 3 # street, place the bet at the end of the three number line.
For a 4 # corner, place the bet in the middle of the square of 4 numbers.
For a 5 # bet, it has to be for 0, 00, 1r, 2b, 3r and is placed at the end of the intersection of the top two lines.
For a 6 # bet, place the bet at the end of the two three number lines at their intersection.
For a 12 # column bet, place the bet at the bottom of the column.
For all other bets, just place your bet in the marked box.

Keep track of your money after each bet:

Start with $ ______ after 1st round _______ after 2nd rd _______ after 3rd rd _______ after 4th rd _______

<table>
<tr><td></td><td></td><td colspan="3">0 | 00</td></tr>
<tr><td rowspan="2">1 to 18</td><td rowspan="4">1st 12</td><td>1 red</td><td>2 black</td><td>3 red</td></tr>
<tr><td>4 black</td><td>5 red</td><td>6 black</td></tr>
<tr><td rowspan="2">Even</td><td>7 red</td><td>8 black</td><td>9 red</td></tr>
<tr><td>10 black</td><td>11 black</td><td>12 red</td></tr>
<tr><td rowspan="2">Red</td><td rowspan="4">2nd 12</td><td>13 black</td><td>14 red</td><td>15 black</td></tr>
<tr><td>16 red</td><td>17 black</td><td>18 red</td></tr>
<tr><td rowspan="2">Black</td><td>19 red</td><td>20 black</td><td>21 red</td></tr>
<tr><td>22 black</td><td>23 red</td><td>24 black</td></tr>
<tr><td rowspan="2">Odd</td><td rowspan="4">3rd 12</td><td>25 red</td><td>26 black</td><td>27 red</td></tr>
<tr><td>28 black</td><td>29 black</td><td>30 red</td></tr>
<tr><td rowspan="2">19 to 36</td><td>31 black</td><td>32 red</td><td>33 black</td></tr>
<tr><td>34 red</td><td>35 black</td><td>36 red</td></tr>
</table>

Standard Roulette Table Layout

18.4) For further problems, we'll focus on conditional probability. Let's assume the ball has already landed on a number, but the spinner is covered. Find the following conditional probabilities that the winning number is:

a) P(4 given that its black), i.e. P(4 | black)

b) P(12 or 14 | even)

c) P(odd given that it's in 2nd 12)

d) P(red | 19 to 36)

e) P(at least 30 given that it's in 3rd 12)

f) P(19 | odd)

g) P(at most 10 given it's black)

h) P(36 | black)

i) P(red given it's 27)

j) P(even | 20)

k) P(in 1st 12 given it's odd)

l) P(black | at least 28)

m) P(in 19 to 36 given that it's divisible by 5)

n) P(3rd 12 | at most 13)

18.5) You place a $20 bet on a street (3 numbers) and the house odds are 11 to 1.

a) What is the probability that you will win?

b) What are the odds against you winning your bet?

c) How much would the casino give you if you win your bet (don't include your bet that gets to stay on the table)?

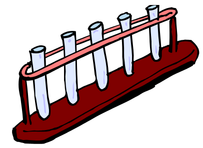
18.6) You place a $50 bet on the middle column (twelve numbers) and the house odds are 2 to 1.

a) What is the probability that you will lose your bet?

b) What are the odds of winning your bet?

c) What would your payout be (not including your bet that stays on the table)?

Group #19



Bayes Theorem

Bayes Theorem goes hand in hand with conditional probability. Before we see the actual theorem, we will look at an example about prenatal genetic testing for Down Syndrome to understand what motivates Bayes Theorem.

Ex. Pregnant women get a first trimester screening which is used to detect Down Syndrome in the fetus. The screening will detect the presence of the birth defect 85% of the time. It is believed that 5% of pregnant women who are NOT carrying a child with Down Syndrome are told they tested positive by the screening. (This is known as a FALSE POSITIVE.)

a) Down Syndrome occurs in about 1 in 1300 pregnancies of a 25 year old woman. If the screening positively detected the presence of the birth defect in a 25 year old pregnant woman, what is the probability that the fetus actually has Down Syndrome? Let DS = has Down Syndrome.

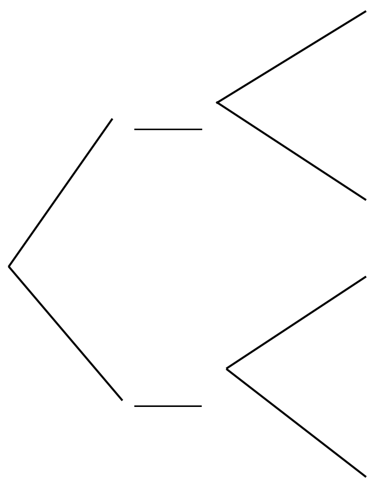
- First determine what the question is asking us.

  Find P(|)

- Next we will determine the following: P(DS) = _______ P(not DS) = _______ P(tests + |DS) = _______

  P(tests − | DS) = _______ P(tests + | not DS) = _______ P(tests − | not DS) = _______

- Make a probability table.



_____ P( )P(|) = ______________

_____ P( )P(|) = ______________

_____ P( )P(|) = ______________

_____ P( )P(|) = ______________

- Now we can determine our answer.

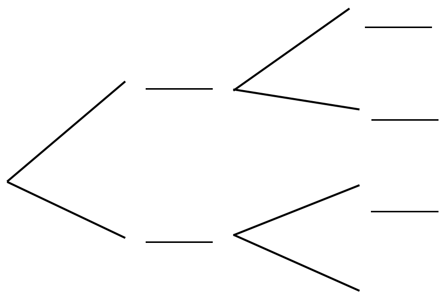
b) For a 45 year old woman the risk goes up to 1 in 35. Determine P(DS | +) for a 45 year old woman.

Bayes Theorem is a fancy way of saying what we just determined using the probability tree. Here is the short form:

BAYES THEOREM $$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)} = \frac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P(A \mid B^C)P(B^C)}$$

19.1) A manufacturer claims that their test will detect steroid use (i.e. positive result for an athlete who uses steroids) 95% of the time. What the manufacturer does not tell you is that 10% of those who do not use steroids test positive (known as a FALSE POSITIVE). On the rugby team, 15% of the athletes use steroids. What is the probability your friend who tests positive actually uses steroids? U = uses, DU = doesn't use.

What are we looking for? P(|)



P( )P(|) = ______________

P( )P(|) = ______________

P( )P(|) = ______________

P( )P(|) = ______________

Determine the answer.

19.2) A test for a genetic disorder can detect with 94% accuracy. The test will incorrectly report positive results for 3% of those individuals without the disorder (a false positive). If 12% of the population has the disorder, find the probability that someone who tests positive has the disorder.

19.3) A fictional casino has two types of slot machines, type A which hit a jackpot 1% of the time and type B that hit a jackpot 10% of the time. Of the casino's 100 slot machines, only 2 of them are type B. If someone hits the jackpot, what is the probability that they were at a type B slot machine?

19.4) Holmes Sports Outlet receives 70% of its track spikes from LA and 30% from NY. The spikes from NY are defective 10% of the time, and those from LA are defective 5% of the time.

a) If a randomly selected pair of spikes is found to be defective, what is the probability that they came from NY?

b) If it is known that a pair of spikes is not defective, what is the probability that they came from NY?

19.5) As winter approaches many college students will get a flu shot. 65% of college students get a flu shot. Of the students getting a flu shot, 15% will contract a flu virus. Of those students *not* getting a flu shot, 72% will contract a flu virus.

a) A college student with the flu virus is selected at random, what is the probability that the student had a flu shot?

b) What is the probability that a student who did not get the flu, did not have a flu shot?

19.6) A biker is doing a ride with a group of friends but gets lost. There are 3 roads at the next intersection. If the biker selects road A, the probability they will find their friends is 1/4. If they select road B, the probability is 1/5, and if they select road C, the probability is 1/6. The probability that the biker chooses any of these roads is the same. If the biker does find his friends, what is the probability that they travelled on road B?

19.7) It is estimated that the NCAA rents 53% of their cars from Agency A, 16% from Agency B, and 31% from Agency C. It is also estimated that 27% of cars from A need a tune up, 12% from B, and 22% from C.

a) What is the probability that a rental car delivered to the NCAA will need a tune up?

b) If a rental car delivered to the NCAA needs a tune up, what is the probability that it came from:

i) A?

ii) B?

iii) C?

19.8) A volleyball game, a basketball game, and a hockey game are being played at the same time. There is a 1/3 chance of the hockey game receiving local television coverage, a 1/2 chance for the basketball game, and a 1/6 chance for the volleyball game. A sports announcer covers 60% of the basketball games, 30% of the volleyball games, and 45% of the hockey games that are televised locally. If the announcer is covering the televised game, which sport is it most likely to be?

Group #20

Pokeno

Pokeno is the combination of Poker and Keno both of which are played in casinos. Everyone gets a card with a 5 by 5 grid of playing cards on it (similar to bingo). In Pokeno, all of the cards are arranged so that each vertical or horizontal line represents a different poker hand, ranging from a royal straight flush to one pair. Before we play Pokeno, we will look at Poker hands so that everyone understands the different types.

20.1) Here is a list of each type of poker hand. Rank the types in order from 1 to 10, 1 being the most probable hand you could have (the lowest ranked hand; every other hand beats it) and 10 being the least probable hand (the highest ranked hand: the hardest one to get; beats every other hand).

_______**Flush**: consists of 5 cards of the same suit
ex. {4♠, 7♠, 10♠, Q♠, K♠}

_______**Two Pairs**: consists of 2 cards of one rank, 2 cards of another rank, and a singleton at a third different rank
ex. {2♥, 2♦, 6♣, 6♥, 9♦}

_______**High Card Only**: consists of cards which fall in no other category, thus there are 5 different nonconsecutive ranks in at least 2 different suits
ex. { A♣, 5♥, 7♥, 8♠, J♠}

_______**Royal Straight Flush**: consists of 10, J, Q, K, A of the same suit
ex. {10♣, J♣, Q♣,, K♣, A♣}

_______**Four of a Kind**: consists of 4 cards of the same rank and one singleton
ex. {8♥, 8♦, 8♠, 8♣, 3♥}

_______**Full House**: consists of 3 cards of the same rank and a pair of cards of another rank
ex. {5♥, 5♠, 5♣, 9♣, 9♦}

_______**One Pair**: consists of 2 cards of one rank, and 3 singletons each of a differing rank
ex. {J♦, J♠, A♥, 4♣, 6♦}

_______**Straight Flush**: consists of 5 cards with consecutive ranks in the same suit (not Royal)
ex. {3♥, 4♥, 5♥, 6♥, 7♥}

_______**Straight**: consists of 5 cards with consecutive ranks in any number of suits
ex. {6♥, 7♠, 8♦, 9♥, 10♦}

_______**Three of a Kind**: consists of 3 cards of the same rank along with two singletons each of a differing rank
ex. {A♣, A♦, A♥, 7♥, 8♠}

20.2) Now everyone take a card and for your card determine which type of poker hand each of the following are:

1st row across: 1st column down:

2nd row across: 2nd column down:

3rd row across: 3rd column down:

4th row across: 4th column down:

5th row across: 5th column down:

diagonal from top left to bottom right: other diagonal:

Your group will now play Pokeno.

1) Have someone turn cards from the deck over one at a time and if you have the card on your Pokeno card, mark it with a penny.
2) Keep going until everyone has a "bingo" (5 in a row). If you get one early on, keep marking because if you get another you can use whichever hand is the best.
3) Once everyone has a "bingo", check who has the highest hand. Ties occur when two hands are identical or if they both have the same straight flush of different suits. In poker, there is not an ordering of suits as there is in other games, such as bridge.

20.3) What is the probability of being dealt a 5 card poker hand with the following properties:

a) all spades?

b) all red?

c) 3 clubs and 2 diamonds?

d) 3 eights and 2 queens?

e) exactly 3 jacks?

f) no hearts?

g) at least 4 red?

h) at most 2 clubs?

i) no face cards?

j) at least one face card?

Group #21

Probabilities of Poker

This project will investigate the probabilities of all of the different types of poker hands which we discussed in Pokeno. One thing to note is that a Royal Straight Flush is also a Straight Flush, a Flush, and a Straight, but you would only consider it a Royal Straight Flush. Once you're done with the project, you will be able to fill out the following table and easily see why the poker hands are ranked as they are. Give probabilities to 4 significant figures (i.e. until you have 4 digits that aren't 0's).

Poker Hand	Number of Each Type of Hand	Probability (4 sig figs)
Royal Straight Flush		
Straight Flush		
Four of a Kind		
Full House		
Flush		
Straight		
Three of a Kind		
Two Pairs		
One Pair		
High Card Only		
Total		

21.1) How many different 5 card poker hands are there? Make sure you have this right because you'll use this number in every other problem!

21.2) A **Royal Straight Flush** consists of 10, J, Q, K, A of the same suit.

a) How many different Royal Straight Flushes are there?

b) What is the probability that you are dealt a Royal Straight Flush?

21.3) A **Straight Flush** consists of 5 cards with consecutive ranks in the same suit, but not Royal.

a) What ranks could be the low card in a Straight Flush?

b) Given your answer in a), how many different Straight Flushes are there?

c) What is the probability that you are dealt a Straight Flush?

21.4) **Four of a Kind** consists of 4 cards of the same rank together with one singleton.

a) How many ways are there to choose the 4 cards that are of the same rank?

b) After those 4 cards have been chosen, how many ways are there to choose the singleton?

c) Given your answers to a) and b), how many different Four of a Kinds are there?

d) What is the probability of being dealt a Four of a Kind?

21.5) A **Full House** consists of 3 cards of the same rank and a pair of cards of another rank. This one is a little more involved, so we'll think about it in pieces…

a) Choose any two ranks ___________ and ____________.

b) Using those two ranks, write out two different Full Houses you could have.

c) How many possible Full Houses could you have using those two ranks? Remember that either rank could be the 3 of a kind or the pair.

d) How many ways are there to choose any two ranks from the 13 in a deck of cards?

e) To figure out the total number of Full Houses, think of choosing your two ranks first (your answer from d)) and from there figure out how many ways there are to arrange those ranks into Full Houses (your answer from c)). How many different Full Houses could you be dealt?

f) So what is the probability that you are dealt a Full House?

21.6) A **Flush** consists of 5 cards of the same suit.

a) How many different hands have 5 cards of the same suit?

b) The answer for a) will not be the right number of Flushes though because Royal Straight Flushes and Straight Flushes have already been counted, so we'll need to subtract them from a). So how many Flushes are there?

c) So what is the probability that you are dealt a Flush?

21.7) A **Straight** consists of 5 cards with consecutive ranks in any number of suits.

a) List the ranks that could be the low card in any straight (include 10 as well). So how many cards could be the low card?

b) After you choose the low card of the straight, how many ways are there to choose each consecutive rank in any suit which will have to be done 4 times?

c) How many hands have 5 ranks in consecutive order?

d) We again have to subtract some that we have already counted, which ones do we need to subtract?

e) How many different Straights are there?

f) And the probability of being dealt a Straight?

21.8) **Three of a Kind** consists of 3 cards of the same rank along with two singletons each of a differing rank, so 3 different ranks are needed. Try to work this one out as we did the Full House: choose the ranks first, then which of those will be the triple, and then the actual cards of each rank.

a) How many different Three of a Kinds are there?

b) What is the probability of being dealt a Three of a Kind?

21.9) **Two Pairs** consists of 2 cards of one rank, 2 cards of another rank, and a singleton at a third rank. Again try to work this one as we did the Full House and the Three of a Kind.

a) How many different Two Pairs are there?

b) What is the probability of being dealt Two Pairs?

21.10) **One Pair** consists of 2 cards of one rank, and 3 singletons each of a differing rank, so 4 ranks are needed. Try with same method.

a) How many different hands have One Pair?

b) What is the probability of being dealt One Pair?

21.11) **High Card Only** When you're playing poker and no one has any of these special poker hands, the hand with the highest card (aces are high) wins and if there's a tie, you move to the 2nd high in those hands, etc. Using the table that you have filled in on the first sheet, you can figure out these answers by knowing the total number of 5 card hands and that probability adds up to 1.

a) What is the number of possible high card only hands?

b) What is the probability of being dealt no special poker hand and have a High Card Only hand?

Group #22

Pinochle Poker

Pinochle is a game which uses a slightly different deck of cards than a standard 52 card deck. We are not going to be learning how to play pinochle but we are going to expand our version of poker using a deck of pinochle cards, so first we will need to get familiar with a pinochle deck.

22.1) Take your pinochle deck and organize the deck in suits and ranks within suits to determine the following.

a) Number of cards in a deck:

b) List the ranks in each suit:

c) Number of cards of each suit:

d) Number of cards of each rank:

e) How many face cards are there?

22.2) a) How many 5 card hands are there in pinochle poker?

b) What new types of poker hands exist when using a pinochle deck?

22.3) A **Royal Straight Flush** consists of 10, J, Q, K, A of the same suit.

a) How many different Royal Straight Flushes are there in hearts?

b) So how many total different Royal Straight Flushes are there in pinochle poker?

c) What is the probability that you are dealt a Royal Straight Flush?

22.4) A **Straight Flush** consists of 5 cards with consecutive ranks in the same suit, but not Royal.

a) How many different Straight Flushes are there in pinochle poker?

c) What is the probability that you are dealt a Straight Flush?

22.5) **Five of a Kind** consists of 5 cards of the same rank.

a) How many different 5 of a Kind are there in pinochle poker?

b) What is the probability that you are dealt a Five of a Kind?

22.6) **Four of a Kind** consists of 4 cards of the same rank together with one singleton.

a) How many different Four of a Kinds are there in pinochle poker?

b) What is the probability of being dealt a Four of a Kind?

22.7) A **Flush** consists of 5 cards of the same suit.

a) How many different hands have 5 cards of the same suit?

b) Remember the answer in a) counts the Royal and Straight Flushes, so you need to subtract to answer: How many Flushes are there in pinochle poker?

c) What is the probability that you are dealt a Flush?

With a pinochle deck, there are 3 types of Flushes: Flush with Two Pairs, Flush with One Pair, and Flush with No Pair, so let's look at each of these separately…

e) Determine the number of ways and the probability of being dealt a **Flush with Two Pairs** in pinochle poker.

f) Determine the number of ways and the probability of being dealt a **Flush with One Pair** in pinochle poker.

g) Determine the number of ways and the probability of being dealt a **Flush with No Pair** in pinochle poker. Hint: Just subtract the above two from the total number of Flushes.

22.8) A **Full House** consists of 3 cards of the same rank and a pair of cards of another rank.

a) How many possible Full Houses are there in pinochle poker?

b) What is the probability of being dealt a Full House?

22.9) A **Straight** consists of 5 cards with consecutive ranks in any number of suits.

a) List the ranks that could be the low card in any straight. So how many cards could be the low card?

b) After you choose the low card of the straight, how many ways are there to choose each consecutive rank in any suit which will have to be done 4 times?

c) How many hands have 5 ranks in consecutive order?

d) We have to subtract previously counted straights. How many different Straights are there in pinochle poker?

e) And the probability of being dealt a Straight?

22.10) **Three of a Kind** consists of 3 cards of the same rank along with two singletons each of a differing rank, so 3 different ranks are needed. Try to work this one out as we did the Full House: choose the ranks first, then which of those will be the triple, and then the actual cards of each rank.

a) How many different Three of a Kinds are there?

b) What is the probability of being dealt a Three of a Kind?

22.11) **Two Pairs** consists of 2 cards of one rank, 2 cards of another rank, and a singleton at a third rank. Again try to work this one as we did the Full House and the Three of a Kind.

a) How many different Two Pairs are there in pinochle poker? Remember to subtract off the Flushes with Two Pairs.

b) What is the probability of being dealt Two Pairs?

22.12) **One Pair** consists of 2 cards of one rank, and 3 singletons each of a differing rank, so 4 ranks are needed. Try with same method.

a) How many different hands have One Pair in pinochle poker? Remember to subtract off the Flushes with One Pair.

b) What is the probability of being dealt One Pair?

22.13) **High Card Only** When you're playing pinochle poker and no one has any of these special poker hands, the hand with the highest card (aces are high) wins and if there's a tie, you move to the 2nd high in those hands, etc.

a) What is the number of possible high card only hands?

b) What is the probability of being dealt no special poker hand and have a High Card Only hand?

22.14) Now let's rank the five card hands in pinochle poker like we did for regular poker. Here's a list of the hands to consider and put in order: Royal Straight Flush, Straight Flush, 5 of a Kind, 4 of a Kind, Full House, Flush with Two Pairs, Flush with One Pair, Flush with No Pair, Straight, 3 of a Kind, Two Pairs, One Pair, and High Card Only.

<table>
<tr><th colspan="2">Pinochle Poker Hand</th><th>Number of Each Type of Hand</th><th>Probability (4 sig figs)</th></tr>
<tr><td rowspan="2">Tie</td><td></td><td>128</td><td>0.00007475</td></tr>
<tr><td></td><td>128</td><td>0.00007475</td></tr>
<tr><td colspan="2"></td><td>336</td><td>0.0001962</td></tr>
<tr><td colspan="2"></td><td>480</td><td>0.0002803</td></tr>
<tr><td colspan="2"></td><td>512</td><td>0.000299</td></tr>
<tr><td colspan="2"></td><td>1920</td><td>0.001121</td></tr>
<tr><td colspan="2"></td><td>16,800</td><td>0.009811</td></tr>
<tr><td colspan="2"></td><td>47,040</td><td>0.02747</td></tr>
<tr><td colspan="2"></td><td>65,280</td><td>0.03812</td></tr>
<tr><td colspan="2"></td><td>130,560</td><td>0.07625</td></tr>
<tr><td colspan="2"></td><td>215,040</td><td>0.1256</td></tr>
<tr><td colspan="2"></td><td>375,840</td><td>0.2195</td></tr>
<tr><td colspan="2"></td><td>858,240</td><td>0.5021</td></tr>
<tr><td colspan="2">Total</td><td>1,712,304</td><td>1.000897 ≈ 1</td></tr>
</table>

22.15) How does this ranking compare to the one for a regular deck of cards? Were there any rankings that surprised you?

Ideas from: www.amstat.org/publications/jse/v20n2/wroughton.pdf

Group #23

KENO

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

10 SPOT GAME

Match	Prize
10	$100,000
9	$5000
8	$500
7	$50
6	$10
5	$2
0	$5

Odds: 1 in 9.05

9 SPOT GAME

Match	Prize
9	$25,000
8	$2000
7	$100
6	$20
5	$5
4	$2

Odds: 1 in 6.53

8 SPOT GAME

Match	Prize
8	$10,000
7	$50
6	$10
5	$2
4	$5

Odds: 1 in 9.77

7 SPOT GAME

Match	Prize
7	$2,000
6	$100
5	$11
4	$5
3	$1

Odds: 1 in 4.23

6 SPOT GAME

Match	Prize
6	$1,100
5	$57
4	$7
3	$1

Odds: 1 in 6.19

5 SPOT GAME

Match	Prize
5	$410
4	$18
3	$2

Odds: 1 in 10.34

4 SPOT GAME

Match	Prize
4	$72
3	$5
2	$1

Odds: 1 in 3.86

3 SPOT GAME

Match	Prize
3	$27
2	$2

Odds: 1 in 6.55

2 SPOT GAME

Match	Prize
2	$11

Odds: 1 in 16.63

1 SPOT GAME

Match	Prize
1	$2

Odds: 1 in 4

Keno is a casino version of the lottery. The casino has a container filled with N balls numbered from 1 to N. A player buys a ticket with which they select anywhere a certain amount of those N numbers and marks them with a "spot." Then the casino or lottery commission selects M winning numbers and you see how many you have matched.

For this group we will be studying Ohio's OH! lottery version of Keno which is played by buying your ticket at a gas station, etc. In OH! Keno you decide how many spots you want to play (1 to 10). Then the casino (or lottery) mechanically (or computer generated program) **chooses 20 winning numbers** from the 80 possible numbers. If a sufficient number of the player's spots are winning numbers, the player receives an appropriate payoff (noted in the table).

23.1) For this problem **three** spots are marked, the player wins if 2 or 3 of their spots are selected. Put an X through 3 numbers on the above table.

a) How many total ways can a player select 3 spots from the 80 numbers?

b)

Outcome	Number of ways	Probability (3 decimal places)
3 matched spots ($27 payout)		
2 matched spots ($2 payout)		
1 or fewer matched ($0 payout)		

c) The lottery card says that the overall "odds" of winning any money in the 3 spot game are 1 in 6.55. These are not actually odds, they are the probability: 1 in 6.55 means 1/6.55 probability. Verify this is true by adding up the number of ways to win money and determine the probability.

23.2) Looking at the "odds" for each game, how many spots gives you the i) best and ii) worst probability of winning something?

23.3) This time **four** spots are marked, the player wins if 2 or more of their spots are selected.
Choose 4 numbers from 1 to 80.

_____ _____ _____ _____

a) How many total ways are there to choose 4 numbers out of 80?

b)

Outcome	Number of ways	Probability (3 decimal places)
4 matched spots ($72 payout)		
3 matched spots ($5 payout)		
2 matched spots ($1 payout)		
1 or fewer matched ($0 payout)		

c) Verify the "odds" of winning some money are 1 in 3.86.

23.4) For this problem **six** spots are marked, the player wins if 3 or more of their spots are selected.
Choose 6 different numbers from 1 to 80:

_____ _____ _____ _____ _____ _____

a) How many total ways can a person choose 6 of 80 #s?

b)

Outcome	Number of ways	Probability (5 decimal places)
6 matched spots ($1100 payout)		
5 matched spots ($57 payout)		
4 matched spots ($7 payout)		
3 matched spots ($1 payout)		
2 or fewer matched		

c) Verify the odds of winning some money are 1 in 6.19.

23.5) POWERBALL In 2007 Powerball was a lotto game played 29 States (including IN), D.C. and the US Virgin Islands. Every Wednesday and Saturday at 10:59 p.m. Eastern Time, 5 white balls were drawn out of a drum with 55 white balls and 1 red ball out of a drum with 42 red balls. There were 9 ways of winning listed in the table. Each ticket costs \$1. The jackpot (match all 5 white balls in any order and the red Powerball) is either an annuitized prize paid out over 29 years (30 payments) or a lump sum payment. On September 18, 2008, www.powerball.com reported that the "Current Estimated Jackpot" is \$154 million or \$78.6 million cash value. Powerball is still played but the numbers have changed as you can see in part d).

a) Determine the total number of different Powerball tickets in a given drawing.

b) Fill in the table. Give the probability of winning in "1 in b" form rounding to a whole number.

Match	Prize	Number of ways	1 in b
○○○○○ + ●	Jackpot		
○○○○○	\$200,000		
○○○○ + ●	\$10,000		
○○○○	\$100		
○○○ + ●	\$100		
○○○	\$7		
○○ + ●	\$7		
○ + ●	\$4		
●	\$3		
○○ or ○ or none	\$0		

c) On www.powerball.com they claimed that the overall "odds" of having a winning ticket is better than 1 in 37. Verify that this is true.

d) Now Powerball has you choose 5 numbers from 1-59 and one Powerball number from 1-35 and a ticket costs \$2. How many different Powerball tickets are there now? Why do you think they changed the numbers?

e) Mega Millions is another lottery which we now have in Indiana. You pick 5 numbers from 1-56 and a Megaball number from 1-46. How many different Mega Millions tickets are there?

23.6) A Keno game has you choose 3 numbers from 1 to 50. Then the lottery commission chooses 10 winning numbers and you win money if you match 2 or 3 of the winning numbers.

a) How many different Keno tickets are there?

b) Fill in the table.

Outcome	Number of ways	Probability (3 decimal places)
3 matched spots ($50 payout)		
2 matched spots ($10 payout)		
0 or 1 matched ($0 payout)		

c) The back of the Keno card says that the “odds” of winning any money are 1 in 10.2. Verify this.

23.7) In a different keno game, you choose 5 numbers from the numbers 1 through 70 and 15 winning numbers are chosen.

a) How many different ways are there to choose your 5 numbers?

b) What is the probability that you match all five numbers?

c) What is the probability that you match 3 of your 5 numbers?

23.8) Yet another keno game has you choose 8 numbers from the numbers 1 through 35 and 10 winning numbers are chosen.

a) How many different ways are there to choose your 8 numbers?

b) What is the probability that you match all 8?

c) What is the probability that you match 3 of the 8?

d) What is the probability that you didn’t match any of your numbers?

Group #24

How much money should you expect to win if you buy a lottery ticket?

In this group we will see how to determine how much money you can expect to win given a lottery ticket and the prizes offered which is also called the **Expectation** or **Expected Value**. The amount that you find to be the expected value will be the average amount of money you could expect to win if you played it over and over and over again.

EXPECTATION is the sum of each outcome multiplied by its probability

$$EV = E_1 \cdot P(E_1) + E_2 \cdot P(E_2) + \cdots E_n \cdot P(E_n) = \sum_{i=1}^{n} E_i \cdot P(E_i)$$

Since we are dealing with games in this course, expectation (also known as expected value) for us will usually mean how much money we can expect to win or lose playing a game. The easiest way to find the expectation for a game is as follows:

1) Make a table with first column consisting of all possible **outcomes**
2) Second column consisting of the **probability** for each outcome
3) Third column with the **product** of the first two (each outcome times its probability).
4) To get the expectation, add up the final column.

A **FAIR GAME** is a game with an expectation of zero. This means if the game is played over and over again, all players are expected to break even (neither lose nor win any money).

Ex1. A game costs $2 to play. You pick a rubber duck from a pond containing 20 of them each with a spot underneath, 1 blue, 3 red, and 16 green. You win $40 if you get the blue, $10 if you get a red, and $0 if you get a green. Find your expected winnings.

Outcomes Probabilities Product

Another way using the cost to play at the beginning:

Outcomes Probabilities Product

Ex2. There are 54 slots on a Big Six Money Wheel. You bet on a denomination and win that amount if it lands on your denomination (plus get your bet back). If a player bets $1 on the $10 denomination, find the player's expectation.

Denominations	# of slots
$40	2
$20	2
$10	4
$5	8
$2	15
$1	23

Ex3. You pay $2 to play the following game. You flip a coin 3 times; if all 3 are heads, you get $7, if 2 are heads, you get $5, if one or none are heads you get nothing.

a) Find your expectation.

b) Find the price of a ticket to make this a fair game.

Now try some expectation problems with your group.

24.1) Looking back at the Keno game from the previous group, determine the expectation for each of the given Spot Games. The price of a ticket is $1.

a) 1 Spot Game

Outcome	Probability	Product
$2 (1 match)	0.25	
$0 (no match)	0.75	

Expectation: ________

b) 2 Spot Game

Outcome	Probability	Product
$11 (2 matches)	0.06	
$0 (0 or 1 match)	0.94	

Expectation: ________

c) What should the price of the ticket be for the 1 Spot Game for it to be fair?

d) What should the price of the ticket be for the 2 Spot Game for it to be fair?

e) Would you play this 1 or 2 Spot Keno game? Why or why not?

24.2) In this keno game, you choose 3 numbers from 1 – 30 and then 10 winning numbers are chosen. You win $10 if you match all 3, $5 if you match 2, and $0 if you match 1 or 0. The price of a ticket is $1.

a) Determine the expected winnings.

Outcome	Probability (leave as fractions)	Product (leave as fractions)
$10		
$5		
$0		

Expectations __________

b) Would you play this game? Why or why not?

24.3) HOOSIER LOTTO The Hoosier Lotto is another lottery, but it is only played in the state of Indiana. For this lottery you choose 6 numbers out of 48 numbers and win if you match at least 2 of the chosen 6. Since 2001 instead of drawing balls, the 6 numbers are chosen by a computer using an RNG or Random Number Generator to avoid any human error.

a) Determine the total number of different Hoosier Lotto tickets in a given drawing.

b) Fill in the the table and then determine the expected value of a $1 ticket for the Hoosier Lotto.

	Outcome		Probability	Product
Match	Prize	Number of winning tickets	1 in b (round to whole #)	3 decimal places
6 of 6	Jackpot (say $1,000,000)			
5 of 6	approx $1000 (pari-mutuel)			
4 of 6	approx $40 (pari-mutuel)			
3 of 6	$3			
2 of 6	Free Quick Pick (worth $1)			
0 or 1 of 6	$0			

Expectation: __________

c) How much money would you expect to win (or lose) if you bought a Hoosier Lotto ticket every day for the next 10 years? Don't worry about leap year.

d) So how much money of the original money you bought the tickets with would you have left after the 10 years?

e) How much $ would you have if you put $1 into a jar in your room every day for the next ten years? Again, don't worry about leap year.

f) Which seems like a better way to use your money?

24.4) You friend Jim has a game for you to play. You roll a fair die. If you roll a 1, Jim pays you \$25. If you roll a 2, Jim pays you \$5. If you roll a 3, you win nothing. If you roll a 4 or 5, you must give Jim \$10, and if you roll a 6, you must give Jim \$15. What is your expected value for this game?

24.5) A game costs \$1 to play. After paying the dollar, we roll a die. If we roll a 1, we get our dollar back (the game is a draw). If we roll a 2 or 3, then we win and are given \$3. If we roll a 4, 5, or 6, then we lose our dollar.

a) What is the expected value of this game?

b) In order to make this a fair game, what should the cost be to play?

24.6) A raffle offers a first prize of \$1000, 2 second prizes of \$300 each, and 20 third prizes of \$10 each. There are 10,000 tickets are sold at \$0.50 each.

a) What are the expected winnings for a person buying one ticket?

b) What would the expectation be if you bought five tickets?

c) What should the price of the tickets be to make this a fair game?

Group #25

Roulette and Expected Value

We will go back to Roulette for this group. Recall the roulette wheel has 18 red spaces, 18 black, and 2 green (considered neither even nor odd). Note: If you win, the bet that you've made goes back to you as well the amount from the house odds. If you lose, you lose the money you bet. If you use the True or Actual Odds, Roulette is a fair game so your expectation will be $0.

Ex. If you place a $1 bet on three numbers (a street) and the casino paid you using the true odds (11.67 to 1), here is how you'd determine the expectation:

Outcome	Probability	Product
Win $11.67	3/38	35/38
Lose - $1	35/38	- 35/38
		EV = $0

You can use the table like above or use the following equation which amounts to the same thing:

($ win)(P(win)) – ($ lose)(P(lose)) = expectation for roulette bet

where odds against are given b to a or $ win to $ lose

Ex. Back to the three number bet, it would look like:

($11.67)(3/38) – ($1)(35/38) = $0

25.1) You place a $1 bet on one number. The house odds are 35 to 1.

a) What is the probability that you will win this bet?

b) How much money would you win if your number comes up (not including your bet that remains on the table)?

c) Determine the expectation for this bet.

25.2) You place a $1 bet on two numbers (a split). The house odds are 17 to 1.

a) What is the probability that you won't win this bet?

b) How much money would you win if either of your numbers comes up (not including your bet remaining on the table)?

c) Determine the expectation for this bet.

25.3) Our example showed that a $1 bet on three numbers (a street) using the true odds made it a fair game. What is the expectation if you use the house odds of 11 to 1?

25.4) You place a $1 bet on four numbers (a corner) with house odds being 8 to 1.

a) What are the odds against you winning this bet?

b) How much money would you lose if your numbers don't come up?

c) Determine the expectation for this bet.

25.5) For that same $1 bet on four numbers, what if the casino used the actual odds of 8.5 to 1, then what would the expected value?

25.6) You place a $1 bet on five numbers which has house odds of 6 to 1.

a) What is the probability that you will win this bet?

b) How much money would you win if your number comes up (not including your bet that remains on the table)?

c) Determine the expectation for this bet.

25.7) Your friend places a $1 bet on a line (six numbers). The house odds are 5 to 1. What is your friends expectation?

25.8) What is the expectation for a $1 bet on twelve numbers (a column) with house odds given as 2 to 1?

25.9) Finally, if you place a $1 bet on either low/high/even/odd/red/black, you're really betting on eighteen numbers. The house odds are 1 to 1.

a) **What is the probability that you will win this bet?**

b) **How much money would you win if your number comes up (not including your bet that remains on the table)?**

c) **Determine the expectation for this bet.**

25.10) Which odds make roulette a fair game? So why do casinos use House odds instead of actual odds?

25.11) Which is the worst bet to make with house odds? How much can you expect to lose for every dollar on this bet? So how much would you expect to lose on a $100 bet?

25.12) Someone has a weighted coin that lands heads up with probability 2/3 and tails up with probability 1/3. If the coin comes up heads, you pay $1; if the coin comes up tails, you receive $1.50.

a) What is the expected value for this game?

b) Would you play? Why or why not?

25.13) Someone has a weighted coin that lands heads up with probability 2/3 and tails up with probability 1/3, but the game has changed. You pay $5 to flip the coin. If the coin comes up heads, you lose your $5. For this game to be a fair game, how much should you receive if you flip tails?

25.14) For several years in Massachusetts, the lottery commission would mail residents coupons for free Lotto tickets. To win the jackpot in MA, you have to correctly guess all six numbers drawn from a pool of 36. What is the expected value of the free Lotto ticket if the jackpot is $8,000,000 and there is no splitting of the prize?

25.15) A friend offers to pay you \$25 if you draw the ace of spades and \$4 for any other spade. However you must pay your friend \$1 for any non-spade drawn. By finding the expected value, determine whether you should accept the offer.

25.16) Now the friend has changed his game and has you grab 2 cards from a deck of cards. If you grab 2 aces he will give you \$100. If you grab 2 face cards, he will give you \$25. Otherwise you will owe him \$2. Find the expected value and determine if you would play this game.

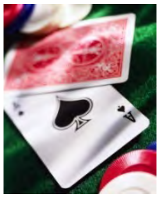
25.17) A school is holding a raffle and sold 900 tickets at \$5 each. The winning ticket holder will get \$500 and 3 runner-up prizes of \$50 each will also be awarded.

a) What are the expected winnings for a ticket?

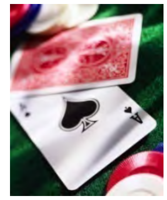
b) What would the tickets cost for this to be a fair raffle?

25.18) A slot machine has 3 wheels each with 10 symbols. On each wheel there is 1 JACKPOT symbol. You put a dollar in the slot and the payoffs are as follows: 3 JACKPOT symbols pays \$487; 2 JACKPOT symbols pays \$10; and 1 JACKPOT symbol pays \$1 (i.e. you get your wager back). Find the expectation for \$1 which will show that this slot machine is fair.

Group #26



Blackjack
A Lesson in Counting Cards



Blackjack is essentially a game between two people – the player and the dealer. To win, a player's hand must have a value closer to 21 than the dealer's hand, without going over 21. Aces are worth either 1 or 11 (at your discretion), tens, jacks, queens, and kings are each worth 10, and all other cards are worth their face values. (The suits of the cards do not matter.) The term "**blackjack**" means that you get a value of 21 with only two cards (an ace and a card with that is worth 10).

- The dealer gives two cards to each player and takes two for him (or her) self (first face down and second face up).
- A game using one or two decks of cards held in the dealers hand is called a "pitch" game and the players cards are dealt face down.
- A game using more decks of cards which are held in a shoe is called a "shoe" game and the players cards are both dealt face up (and they must not touch their cards). This is the way casinos play Blackjack.
- Each player in turn may "hit" – take another card – until they are satisfied with their total or they "bust" with a total exceeding 21. If a player busts, they lose.
- The dealer then plays out their hand following certain rules.
  - The dealer must "hit" if they have 16 or less.
  - The dealer must "stay" (don't take a card) if they are 17 or higher.
- If the dealer busts, they pay off all players with hands still alive.
- If the dealer reaches 17 or better, they pay off players with higher totals less than or equal to 21.
- The only real advantage for the house is that players have a chance to bust first and if both the player and dealer bust, the house wins.

For the first two problems, we will assume there is only one deck of cards being used.

26.1) For the dealer, the order of the cards matters because only one card faces up – for example, the hands {Q, A} and {A, Q} would be considered different hands.

a) Why would it matter if the Ace is up and the Queen down or the Queen up and the Ace down?

i) If the dealer is dealt an Ace up, what is the probability that they have a blackjack? (Assume these are the only cards dealt from the deck.)

ii) If the dealer is dealt a Queen up, what is the probability that they have a blackjack? (Assume these are the only cards dealt from the deck.)

iii) So now we can answer the original question: Why would it matter if the Ace is up and the Queen down or the Queen up and the Ace down?

b) How many different pairs of cards can the dealer be dealt (one dealt up and one dealt down)?

26.2) For players, order wouldn't matter because they're either both face up or both face down.

a) How many different pairs of cards can a player be dealt?

b) How many different Blackjacks are there?

c) What is the probability of being dealt a blackjack?

26.3) In most casino blackjack games, several decks of cards are used and stored in the shoe.

a) Do you think the probability of being dealt a blackjack on the first two cards will increase, decrease, or stay the same if more than one deck of cards is used?

b) Calculate the probability of being dealt a blackjack with the first two cards using: (go to 4 significant figures) Remember you need to figure out how many cards there are, how many aces there are and how many cards worth 10 in the number of given decks.

i) two decks of cards.

ii) three decks of cards.

iii) six decks of cards (number commonly used at casinos).

PLAY BLACKJACK with your group. Have one member be the dealer for the first round and then rotate who deals. Since you are only using one deck (a pitch game) deal the cards to each player face down but remember that the dealer always gets one face down and one face up.

26.4) A **SOFT** hand refers to a hand where an ace is used as an 11 and which can't be busted by taking one more card. For example, a soft 17 is an Ace with a 6. If you hit and get a 5 (or higher), you wouldn't bust at 22, you would instead have 12 and use the Ace as a 1. On the table in front of the dealer, there will be one of the signs. Either it will say "Dealer must draw to 16 and stand at all 17s" meaning even at soft 17s. Or there will be a sign saying, "Dealer must hit soft 17s." These will vary from casino to casino, maybe even table to table.

a) How many different soft 17s could the dealer be dealt with their first two cards (one dealt up and one dealt down)?

b) Disregarding the order dealt, a soft 17 with 3 cards would be one of the following: Ace, Ace, 5; or Ace, 2, 4; or Ace, 3, 3. List all of the possible soft 17s with 4 cards.

c) List all possible soft 17s with 5 cards.

d) If there are six decks of cards in the shoe, what is the largest number of cards you could have in a soft 17? Think of how many aces there are to use.

BASIC STRATEGY

You can increase your odds of winning in blackjack by memorizing the table to the right. It is not illegal or frowned upon; any good blackjack player should know it.

The left column shows what you have in your hand, either by sum (for the first rows) or by the actual combination of cards.

The top row is the one face up card that the dealer has.

You match up what you have and what the dealer is showing to determine what you should do.

H = Hit

D = Double Down (take only one card and double your bet)

S = Stand

P = Split

Your	Dealer's Card									
Hand	2	3	4	5	6	7	8	9	10	A
8	H	H	H	H	H	H	H	H	H	H
9	H	D	D	D	D	H	H	H	H	H
10	D	D	D	D	D	D	D	D	H	H
11	D	D	D	D	D	D	D	D	D	H
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
16	S	S	S	S	S	H	H	H	H	H
17	S	S	S	S	S	S	S	S	S	S
A,2	H	H	H	D	D	H	H	H	H	H
A,3	H	H	H	D	D	H	H	H	H	H
A,4	H	H	D	D	D	H	H	H	H	H
A,5	H	H	D	D	D	H	H	H	H	H
A,6	H	D	D	D	D	H	H	H	H	H
A,7	S	D	D	D	D	S	S	H	H	H
A,8	S	S	S	S	S	S	S	S	S	S
2,2	P	P	P	P	P	P	H	H	H	H
3,3	P	P	P	P	P	P	H	H	H	H
4,4	H	H	H	P	P	H	H	H	H	H
5,5	D	D	D	D	D	D	D	D	H	H
6,6	P	P	P	P	P	H	H	H	H	H
7,7	P	P	P	P	P	P	H	H	H	H
8,8	P	P	P	P	P	P	P	P	P	P
9,9	P	P	P	P	P	S	P	P	S	S
10,10	S	S	S	S	S	S	S	S	S	S
A,A	P	P	P	P	P	P	P	P	P	P

COUNTING CARDS

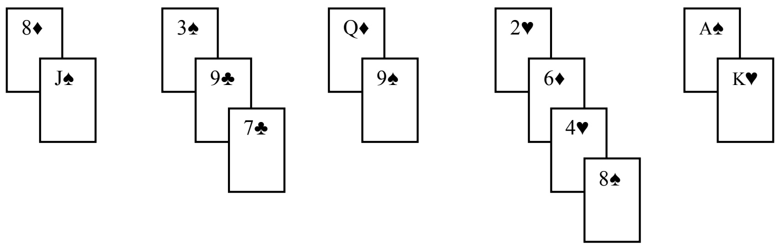
A strategy to improve your odds of winning is the Hi-Lo counting card system which, contrary to common belief, is not illegal. However if a casino notices you doing it, they may have the dealer shuffle the deck more often or even ask you to leave. The strategy is actually quite simple; you just need to pay attention and not get distracted.

You start with your count N = 0 and then add or subtract from it according to:

+ 1 for each card from 2 – 6
0 for each card from 7 – 9
– 1 for each Ace, 10, Jack, Queen, King

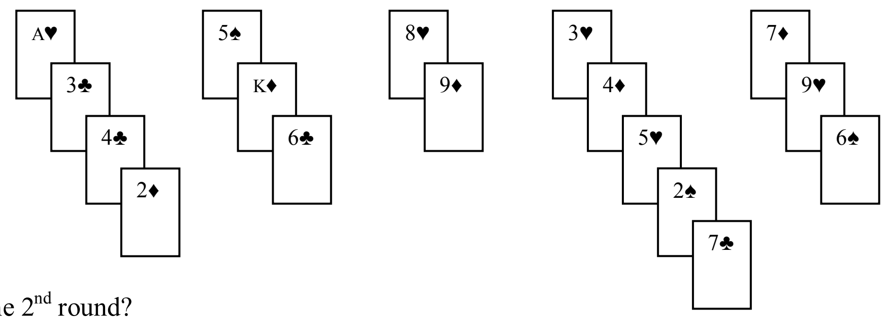
26.5) Try this example: Start with N = 0

1st round:



a) What is N after the 1st round?

2nd round:



b) What is N after the 2nd round?

N will give you an idea of how many high or low cards are left in the deck- the higher N is the more likely there are high cards left and the more likely the dealer will bust and the lower N is the more likely low cards are left. Most casinos use six decks of cards so you need to figure out about how many high or low cards are left in the shoe, so once you get N, you need to divide it by your estimate of the number of decks left in the shoe.

T = N / approximate # of decks left in the shoe

Then your value for T will help you decide how to bet:

$T < 0$ → no bet
$0 < T < 2$ → minimum bet
$T > 2$ → bet the maximum

Back to our example:

c) Say you estimate that there are 5 decks of cards left in the shoe, what is T and how should you bet?

d) If you estimate there are only 2 decks left, what is T and how should you bet?

REVIEW PROBLEMS FOR TEST 2

1) In a game of Let's Make a Deal, the host offers you a choice of 6 doors, five with a goat behind them and one with a car behind it. The host will then open 4 of the remaining doors which reveal 4 goats and ask you if you want to stay with the door you chose or switch to the other door.

a) P(car if you stay) b) P(car if you switch) c) Should you stay or switch?

2) Select two cards from a standard deck.

a) P(both face cards) b) What are the odds of getting two face cards?

3) What are the odds of drawing a five from a standard deck?

4) One card is drawn from a standard deck. What are the odds it's a club or a jack?

5) The odds are 6:1 against it being sunny today. What's the probability that it will be sunny?

6) Out of 250 students interviewed at a community college, 90 were taking math but not chemistry, 160 were taking math, and 50 were not taking either one. Find the probability that a student chosen at random was:

a) taking just chemistry b) taking math or chemistry, but not both

c) taking chemistry d) not taking math

e) taking math, given that the student was taking chemistry

f) taking chemistry, given the student was taking math

g) taking math, given the student was taking chemistry or math

h) taking chemistry, given the student was not taking math

i) not taking math, given the student was not taking chemistry

7) Two dice are tossed. Determine the following.

a) P(sum of #'s is 9)

b) P(dice show same #)

c) P(dice show different #s)

d) sum of #'s is 3 or 7

e) P(doubles | sum of 8)

f) P(one die is a 3 | sum of 9)

8) In roulette, you bet on 5 numbers and the house odds for your payout are 6 to 1. (There are 38 spaces on a roulette wheel.)

a) What is the probability that you will win your bet?

b) If you win your bet, how much money will you get (not including the fact that your bet will stay on the table)?

c) What is your expected payout on a $1 bet?

d) The true odds are 6.6 to 1. What should your expected payout be for a $1 bet using the true odds?

9) Two letters are chosen at random from NUMERICAL. What's the probability that both are vowels?

10) What is the probability of hitting one jackpot on a slot machine with 3 reels each with 10 different symbols (one being a jackpot on each reel)?

11) A coin is tossed five times. What's the probability of getting:

a) all heads?

b) exactly one tail?

c) exactly three heads?

d) at least one head?

12) Four cards are drawn at random from the thirteen hearts in a standard deck. What's the probability the selection contains:

a) no face cards?

b) both king and queen?

c) queen but not king?

d) at least two face cards?

13) A letter is chosen at random from the alphabet. What's the probability that the letter is in the word "house" or "phone"?

14) Two marbles are drawn at random from a bag containing 3 red, 5 blue, and 6 green marbles. What's the probability of drawing:

a) no blue?

b) at least 1 blue?

c) two of the same color?

d) two of different colors?

15) A committee of six people is to be selected from a group of seven men and eight women. What's the probability that the committee contains:

a) all men or all women?

b) five men or five women?

c) at least one woman?

16) A bag contains five pennies and ten nickels. If three coins are selected at random, what's the probability of getting \$0.11?

17) Five cards are drawn at random from a standard deck. Find the probability that:

a) exactly 2 are hearts.

b) all are kings and queens?

c) all red cards?

d) at least 3 aces?

18) A game costs $1 to play. You flip a coin 5 times in a row and count how many heads you have. If you get 5 heads, then you win $10. If you get 3 or 4 heads, you get $5. If you get anything else, you get nothing!

a) Find the expected value of this game.

b) Would you play this game? Why or why not?

19) In a special lottery, 50,000 $1-tickets are sold. The first prize is $15,000 and the five second-prize winners will share $2000 equally. Gerta Gambler buys one ticket. What is her mathematical expectation?

20) In the game of Risk, if the attacker rolls: 5, 4, 3 and the defender rolls: 4, __. List the outcomes as well.

a) P(attacker losing 2 armies)

b) P(defender losing 2 armies)

c) P(both losing one army)

21) State what type of poker hand the following are:

a) 8♣, J♣, Q♦, 9♥, 10♦

b) 7♥, 7♣, J♠, 7♦, 9♥

c) A♠, 4♠, 7♠, Q♠, K♠

22) Five cards are drawn from a 48 card pinochle deck. Recall there are 2 of each of the following ranks in each suit: 9, 10, J, Q, K, A. Determine the following.

a) P(3 jacks)

b) P(five of a kind)

23) In a keno game you pay $1 and choose 3 numbers from the numbers 1 - 70. Then the casino chooses 15 winning numbers and you see how many you have matched.

a) How many total ways can you select 3 numbers from the 70 numbers?

b) Fill in the table and determine the expectation.

Outcome	Number of ways to win	**Probability** (3 decimal places)	**Product**
3 matched ($20 payout)			
2 matched ($2 payout)			
1 or 0 matched ($0 payout)			

Expectation _______________

c) The casino claims that the overall "odds" of winning any $ are 1 in 8.84. Verify this claim.

24) Find the probability of getting each type of poker hand when dealt 5 cards.

a) P(Straight Flush)

b) P(One Pair)

25) A new medical test can determine whether or not a human has a disease that affects 5% of the population. The test correctly gives a positive result for 99% of the people who have the disease, but gives a false positive for 3% of people who do not have the disease. If a person test positive, what is the probability that the person has the disease?

FINAL REVIEW

I. Logic
  A. Puzzles
  B. Sudoku

II. Sets
  A. Definitions (union, intersection, number of subsets, etc)
  B. Venn Diagrams

III. Combinatorics
  A. Order matters- Permutations
  B. Order doesn't matter- Combinations
  C. Coins
  D. Deck of Cards, etc.
  E. Tournaments

IV. Probability
  A. Odds
  B. Conditional probability (B given A happened)
  C. Complement: P(at least one) = 1 – P(none)
  D. Independence (does one event affect another)
  E. Monty Hall
  F. Pair of Dice
  G. Risk
  H. Roulette
  I. Slot Machines
  J. Lotteries (Keno)
  K. Poker 5 card hands
  L. Pinochle 5 card hands
  M. Expectation / Expected Value
  N. Blackjack

FINAL REVIEW PROBLEMS

1) At the local butcher's shop, there were five customers in the lineup. Each of the customers bought something different. The first names of the customers were Annie, Jessica, Lily, Maggie and Naomi. Their last names were Bore, Hazlitt, Piggott, Sowter and Trotter. The available products were: Cumberland sausage, pork chops, pork pie, scotch eggs, and sliced ham.

Lily Piggott was served later than the customer who requested the sliced ham, but before Mrs. Sowter.
The second customer was Maggie.
The pork pie was purchased by the customer directly after Jessica.
Naomi was the woman who bought the scotch eggs; she was served sometime after Annie.
The Cumberland sausage was requested by Mrs. Trotter.
Mrs. Hazlitt was the third in line.
The fourth customer in the line bought the pork chops

a) What was purchased by the third person in line?

b) What was the last name of the person who purchased the pork pie?

c) What place was Naomi in line?

2) There are five houses in a row and in five different colors. In each house lives a person from a different country. Each person drinks a certain drink, plays a certain sport, and keeps a certain pet. No two people drink the same drink, play the same sport, or keep the same pet. Your task is to solve the question “who owns the fish?” with the following clues:

The Brit lives in a red house
The Swede keeps a dog
The Dane drinks tea
The green house is on the left of the white house (when looking at the houses from the street)
The green house owner drinks coffee
The person who plays polo rears birds
The owner of the yellow house plays hockey
The man living in the house right in the center drinks milk
The Norwegian lives in the first house
The man who plays baseball lives next to the man who keeps cats
The man who keeps horses lives next to the one who plays hockey
The man who plays billiards drinks beer
The German plays soccer
The Norwegian lives next to the blue house
The man who plays baseball has a neighbor who drinks water.

	House #1	House #2	House #3	House #4	House #5
country					
color					
drink					
sport					
pet					

3) Fill in the Sudoku.

	4		2	8		1		
	9		1				5	3
		6	4			7		
					9	5	4	8
2				1				6
9	5	8						
1		7			6	3		
4	3				2		7	
6				7	1		9	

4) What are the possible outcomes in a best of 3 series between teams A and team B?

5) Given: $U = \{e, f, g, h, i, j, k, l, m, n, o, p\}$, $A = \{f, h, i\}$, $B = \{k, l, p\}$, and $C = \{i, k, l, m, n, o, p\}$.

a) TRUE or FALSE $i \in C$

b) TRUE or FALSE $A \subseteq B$

c) TRUE or FALSE $\varnothing \subseteq A$

d) TRUE or FALSE $B \subseteq C$

e.) TRUE or FALSE $A = \{h, i, j\}$

f) TRUE or FALSE $\{f\} \in A$

g) n(C) = ________________

h) $\varnothing^c =$ ________________

i) $A \cap B =$ ________________

j) $A \cup B =$ ________________

k) $C^c =$ ________________

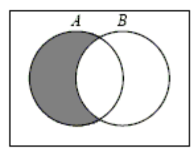
l) $U^c =$ ________________

m) Find $B \cup (A \cup C)^c$

n) How many subsets does set C have?

o) List the subsets of B.

6) Use set notation to describe the shaded area in the Venn diagram.

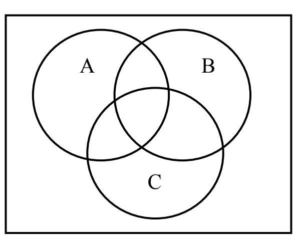


7) If U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }; R = { 2, 4, 6, 8 } and S = { 3, 5, 7 }, then determine $R^c \cap S^c$.

8) Shade the region on the Venn Diagram and state in words what they represent if A = those who eat apples, B = those who eat bananas, and C = those who eat cantaloupe.

a) $(A \cup C)^c$

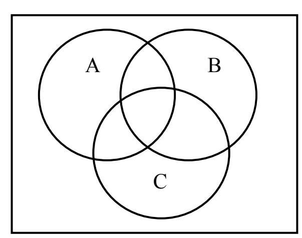
i)



ii) describe

b) $B^c \cap C$

i)

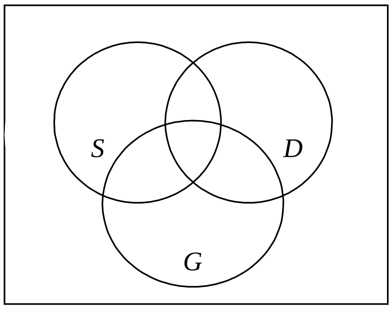


ii) describe

9) Suppose $n(A) = 32$, $n(B) = 91$, and $n(A \cup B) = 100$, find $n(A \cap B)$.

10) Dr. Holmes surveyed the 38 students in her class about their families. 28 students have a brother or a sister, 15 students have a brother, and 8 students have both a brother and a sister. How many students have only a sister?

11) An activities director for a cruise ship has surveyed passengers. Of the 280 passengers, fill in each area of the Venn diagram with the appropriate number of people letting S = swimming, D = dancing and G = games



135 like swimming
80 like games
50 liked dancing but *not swimming*
70 like swimming and dancing *but not games*
45 like swimming and games
25 like dancing and games
10 like all three

a) How many liked either swimming or games?

b) How many liked swimming and dancing?

c) How many liked 2 of the 3 types of activities?

d) How many liked games and dancing but not swimming?

12) How many different license plates are possible if they consist of 3 letters followed by 2 digits if repetition is not allowed?

13) From a group of 6 men and 3 women, how many committees of 2 men and 2 women can be formed?

14) A photographer wishes to display four snapshots in a row picked from nine which they have decided are their best. How many ways can this be done?

15) In how many ways can six people be selected from a group of twelve people?

16) Four married couples attend a concert. How many ways can the couples be seated if each couple must be seated together?

17) Given the probability of winning a soccer game is 8/13, what are the odds that you'll lose the game?

18) If you flip a coin 8 times, what is the probability of getting at least 6 heads?

19) A single card is drawn from a deck. What's the probability of selecting

a) a spade or a 3?

b) 7 or a face card?

c) a Jack or a red?

d) a 10 given it's not a face card?

e) red given it's not a club?

20) The morning line odds *against* High Limit to win the Kentucky Derby are 10 to 1. What is the *probability* that High Limit *will win* the Derby?

21) Five cards are drawn from a standard deck. What is the probability of getting

a) exactly one queen?

b) at least one diamond?

c) two pairs?

22) A deck of pinochle cards consists of 48 cards with 12 cards of each suit, consisting of two 9's, two 10's, two J's, two Q's, two K's, and two A's. Five cards are drawn from a pinochle deck. What is the probability of getting:

a) exactly one queen?

b) at least one diamond?

c) all two pairs (including the flushes with 2 pairs)?

23) 100 students are surveyed; 50 say they like math, 30 say they like English, and 20 say they like both. What's the probability a student likes English given they like math?

24) A pair of dice is tossed. a) P(sum is less than 5) b) P(one die is 3 | sum is less than 5)

25) If three marbles are drawn from 5 blue, 3 red, and 6 white, what is the probability that all three are blue?

26) There are 38 slots on a roulette wheel. You place a $1 bet on a corner (4 numbers).

a) What is the probability that you will win your bet?

b) What are the odds against you winning your bet?

c) The house odds are 8 to 1. What are your expected winnings using the house odds?

d) You bet $10 on a corner. Using the house odds, how much money would you net if you win your bet?

27) In a game, a coin is flipped twice. If heads lands up either time, the player wins $4. If they flip tails twice, the player loses $2. What is the expected value of this game?

28) A hat contains 28 names, 11 of which are male. If three names are randomly drawn from the hat, what is the probability that at least one male name is drawn?

29) A bag consists of the following: 5 red marbles, 3 yellow marbles, and 7 green marbles.

a) What is the probability that when you reach in and grab 3 balls that you get 2 red and 1 yellow?

b) What is the probability that when you grab 3 marbles one at a time without replacement that you choose a red, yellow and then another red?

30) A pharmaceutical company has developed a test for a condition that is present in 20% of the population. The test is 90% accurate in determining a positive result for those who have the condition and the chance that a person who doesn't have the condition is given a positive result 5% of the time (known as a false positive). Determine the probability that someone who tests positive actually has the condition?

31) You are playing a card game with a standard 52 card deck. You pay \$2.00 to play the game. If you choose a red face card you will win \$6.00, if you choose a black face card you will win \$4.00, if you choose any other card, you will win nothing. Calculate your expected winnings.

32) A lotto game lets you pick 5 numbers from the numbers 1 through 30 and then 10 winning numbers are chosen.

a) How many different lottery tickets can be chosen?

b) P(match all 5)

c) P(match 3 of the 5)

33) A slot machine has 5 reels each with 10 symbols one of which is a jackpot.

a) How many outcomes are possible?

b) P(no jackpots)

c) P(3 jackpots)

d) What is the probability that you will at least one jackpot?

34) Monty Hall offers you a choice of 3 doors, one has a car behind it and two have goats behind them. After you choose, Monty will open a remaining door with a goat behind it and then offer you to stay with your original door or swtich. What would you do and why? Use probability.

SOME SOLUTIONS

Group 2 2.5)a)174 b)629 c)052

2.6)	Brenda	James	Diane	Simon
Paint	**Blue**	**Yellow**	**Green**	**Red**
T-shirt	**Yellow**	**Green**	**Blue**	**Red**

2.7)

Place	1^{st}	2^{nd}	3^{rd}	4^{th}
First name	**Anne**	**Mimi**	**Jane**	**Lucy**
Last name	**Guest**	**Joseph**	**Brown**	**Scott**

Group 3 3.4)a)2 b)ii c)1 d)3;5 3.5)a)iv b)J, N c)P, M, L d)ii e)v 3.6)a)B b)D c)C d)D e)A 3.7)a)C b)D

Group 4 4.4)

Bob	Master Key	Bedroom	60
Betty	Now and Then	$0	20
Sue	Cliff Hangers	trip	40
Tom	Dice Game	car	50
Lindsay	Plink	$10,000	30

4.5)

	gold	silver	bronze
France	**5**	**7**	**4**
China	**6**	**4**	**9**
Austria	**8**	**9**	**5**
S Korea	**7**	**3**	**3**

4.6)

level	1	2	3	4	5	6	7	8	9
challenge	**movin on up**	**bobblehead**	**sticky**	**balance**	**egg tower**	**keep it up**	**bottoms up**	**backflip**	**uphill battle**
time	**32**	**27**	**46**	**39**	**43**	**41**	**58**	**52**	**60**
object	**cups**	**pedometer**	**bread**	**salt**	**paper towel**	**feathers**	**yo-yo**	**pencils**	**marbles**

Group 5 HW: 5.1)

Name	Piece	Place
Aaden	**Iron**	**Baltic**
Rachel	**Thimble**	**Kentucky**
Karen	**Ship**	**Pennsylvania**
Colin	**Cannon**	**Boardwalk**
Laura	**Dog**	**Marvin Gardens**
Scott	**Racecar**	**Jail**
Joel	**Wheelbarrow**	**State**
Billy	**Moneybag**	**B & O RR**

5.2)

1892	1915	1930	1959	1978	1994	1996	2007
James	**Jess**	**Max**	**George**	**Muhammad**	**Mike**	**Sultan**	**Ingemar**
Corbett	**Willard**	**Schmeling**	**Foreman**	**Ali**	**Tyson**	**Ibragimov**	**Johansson**
25	**35**	**70**	**81**	**61**	**58**	**24**	**28**
16	**26**	**56**	**76**	**56**	**50**	**22**	**26**

SETS AND VENN DIAGRAMS: 1)a){1,2,3,4,5,7} b)Ø c){1,6,7,8,9} d)5 e){6,8,9} f)F g)T h)F 2)a){Kim} b){A,B,E,J,K,S} c){B,D,E,J} d){A,D,S} e){A,B,E,J,K,S} f){K} g){A,S} h){B,E,J,K,D} i){D} 3)a){red} b){blue,green,orange,yellow,purple,white} c){red,orange,blue} d){red,blue,green,orange,white} e)orange,white} f){yellow,purple} 4)a)3 b)20 c)13 d)10 e)10 f)15 g)22 h)5 5)a)440 b)700 6)10 7 4)a)40 b)16 c)19 d)35 e)81 8)a)60 b)23 c)40 d)72 9)a)19 b)49 c)57 d)61 e)76 10)a)100 b)30 c)20 11)a)450 b)190 c)140 d)95

Group 7: 7.2)a)16 b)8 c)32 d)1 e)8 f)3 g)4 h)6 i)12 j)0 k)26 l)12 m)6 n)26 7.3)e)2, 4, 8, 16 f)2^N 7.4)a) 8 b)256, so yes c)10 d)14 7.5)d)i)those who do not like both solitaire and poker $(S \cap P)^C$ ii)those who like solitaire or bridge $S \cup B$ iii)those who like bridge but not solitaire or poker $B \cap (S \cup P)^C$ 7.6)a)$5 \cup$face,16 b)$\heartsuit \cap$face,22 c)$(\text{red} \cup \clubsuit)^C$,13 d)$\text{red} \cap J^C$,24 e)$(\text{face} \cap \text{black}) \cap Q^C$,4 f)$(3 \cup \spadesuit) \cap \heartsuit^C$,15 7.7)a)$B \cup S$ b)P^C c)$B \cap P^C$ d)like poker and bridge, $P \cap B$ e)like none of the three games,$(S \cup B \cup P)^C$ f)don't like all 3 games,$(P \cap B \cap S)^C$

Group 8: 8.1)1 8.2)9 8.3)8 8.4)225 8.5)a)72 b)47 c)25 d)0 8.6)5 8.7)3 8.9)a)342 b)192 c)72 d)86 8.10)a)35 b)16 c)7 d)0 8.11)a)275 b)78 c)16 8.12)a)22 b)50 c)11 d)25 e)11 8.13)16

Group 9: 9.1)i)play L, S, and B ii)play L and S but not B iii)play L and B but not S iv)play S and B but not L v)play only L vi)play only S vii)play only B viii)plays none of the 3 9.2)i)$L\cap S\cap B$ ii)$L\cap S\cap B^C$ iii) $L\cap B\cap S^C$ iv) $S\cap B\cap L^C$ v)$L\cap(S\cup B)^C$ vi)$S\cap(L\cup B)^C$ vii)$B\cap(L\cup S)^C$ viii)$(L\cup S\cup B)^C$ 9.6)$\{2,4,b,d\}$ 9.7)$\{d,f,h\}$ 9.8)b)$\{1,3,5,7,9,10\}$ c)$\{2,3,5,7\}$ d)$\{6,7,8,9\}$ 9.9)b)$\{3,4,5,8\}$ c)$\{2,5,6,8\}$ d)$\{3,6,7,8\}$

INTRO TO COMBINATORICS: 1)a)125 b)60 c)24 2)160 3)12 4)12 5)256 6)12 7)100,000 8)2646 9)512 10)96 11)90 12)5832 13)120

Group 11: 11.1)a)3 b)6 c)6 d)3 e)1 f)2 11.2)a)4 b)12 c)24 d)24 e)6 f)4 g)1 h)6 11.3)a)5 b)1 c)3 d)6 e)10 f)3 g)6 h)10 i)5 j)1 11.4)a)i)720 ii)1000 iii)360 b)i)5040 ii)10,000 iii)5000 c)i)30,240 ii)100,000 iii)336

PERMUTATIONS AND COMBINATIONS: 1)133,784,560 2)3,628,800 3) 924 4)a)924 b)180 c)378 d)462 5) a)8008 b)2002 6)a)91 b)286 c)66 7)23,523,786 8)65,780 9)24 10)a)210 b)35 11)a)2 b)6 12)a)120 b)12 c)24 d)12 13)376,992 14)22,957,480 15)5/36 16)91 17)a)7,880,400 b)1,313,400 18)360 19)a)362,880 b)288 c)1728 20)2916

Group 12: 12.1)20 12.2)120 12.3)336 12.4)35 12.5)120 12.6)15 12.7)40,320 12.8)259,459,200 12.9)165 12.10)2,598,960 12.11)324 12.12)8 12.13)4 12.14)2652 12.15)720 12.16)6 12.17)1000 12.18)286 12.19)60 12.20)40 12.21)970,200 12.22)$2.788\text{x}10^{10}$ 12.23)24 12.24)8008 12.25)42 12.26)105 12.27)24 12.28)64,350 12.29)15 12.30)60 12.31)a)220 b)84 c)6 d)720 e)140 f)40 12.32)a)79,833,600 b)604,800 c)161,280

Group 13: 13.1)2,598,960 13.2)65,780 13.3)65,780 13.4)1287 13.5)845,000 13.6)575,757 13.7)202,800 13.8)22,308 13.9)9295 13.10)712,842 13.11)27,885 13.12)454,480 13.13)2,357,862 13.14)29,172 13.15)24 13.16)4 13.17)48 13.18)2,490,624 13.19)4512 13.20)792 13.21)1,712,304 13.22)886,656 13.23)652,080 13.24)624

Group 14: 14.1)a)3 b)7 c)11 d)N-1 14.2)a)2 or 3 b)4 or 5 c)6 or 7 d)2N-1 or 2N-2 14.3) SS, SDS, SDD, DD, DSS, DSD 14.4) SSS, SSDS, SSDDS, SSDDD, SDSS, SDSDS, SDSDD, SDDSS, SDDSD, SDDD, DDD, DDSD, DDSSD, DDSSS, DSDD, DSDSD, DSDSS, DSSDD, DSSDS, DSSS 14.5)a)3 b)6 c)10 d)NC2 14.7)64 14.8)67 14.9)134 or 135

TEST 1 REVIEW: 1)1st Ben J fruit, 2nd James B chocolate, 3rd Nigel S sponge, 4th Vicky A cheese 2)$A\cap B^C$ 3)a)T b)$\emptyset,\{7\}$ c)T d)$\{3,5\}$ e)16 f) $\emptyset$ g)$\{1,3,4,5,6,7\}$ h)T i) $\subseteq$ j)F k)F l)F m)$\{3,5\}$ n)6 4) 190 5)a)1130 b)1910 c)950 d)280 6)all of C except the very middle plus only A 8)a)4 b)6 c)Z and Y 9)a)990 b)165 10)a)1287 b)264 c)454,480 11)a)924 b)180 c)378 d)462 12)144 13)a)216 b)120 c)80 14)503 15)a)13 b)26 or 27 16)a)0 b)16 c)15

PROBABILITY PROBLEMS: 1)a)46,656 b)0.0000214 2)10/32 3)a)11/75 b)59/75 4)11/26 5)3/11 6)a)Y:2/3, RS:5/7 b)Red Sox 7)a)5/14 b)6/7 c)1/31 8)a)2/30 b)2to28 9)12/50 10)a)1/45 b)17/45 11)3/21 12)4/22,100 13)18/27 14)a)6/120 b)10/120 c)21/120 d)66/120 e)54/120 f)83/120 15)671/1296 16)32/51 17)a)1/1947792 b)1/3,838,380 18)a)72/1326 b)376/1326 c)330/1326 19)a)5005/177,100 b)210/177,100 c)22050/177,100 d)54600/177,100 e)35035/177,100 f)172095/177,100 20)29/128

Group 15: 15.4)1/3 15.5)2/3 15.6)switch 15.8)a)1/3 b)1/3 c)2/3 d)2/3 15.11)a)1/4 b)3/4 c)switch 15.12)a)i)1/5 ii)4/5 b)i)9/10 ii)1/10 c)i)1/N ii)(N-1)/N 15.13)1/4; switch after 2nd offer 15.14)a)4:1 b)1:7 c)7:18 15.15)a)14:3 b)7:3 c)1:4 15.16)a)5/6 b)3/5 c) 9/14 15.17)a)5/9 b))1/10 c)9/11 15.18)48:4 15.19)84:16 15.20)2/11 15.21)3:5 15.22)4:32

Group 16: 16.1)36 16.2)a)6/36 b)1/36 c)2/36 d)3/36 e)4/36 f)5/36 g)6/36 h)5/36 i)4/36 j)3/36 k)2/36 l)1/36 m)5/36 n)10/36 o)10/36 16.3)a)6;1/6 b)1,2;2/6 c)4,5,6;3/6 16.4)a)0 b)3,4,5,6;4/6 c)1,2;2/6 16.5)a)1;1/6 b)4,5,6;3/6 c)2,3;2/6 16.6)a)0 b)1,2,3,4,5;5/6 c)6;1/6 16.7)a)1,2;2/6 b)5,6;2/6 c)3,4;2/6 16.8) a)3,4,5,6;4/6 b)0/6 c)1,2;2/6 16.9)a)8/36 b)15/36 c)13/36 16.10)a)8/36 b)4/36 c)24/36 d)6/36 e)30/36 f)6/36, no 16.11)a)30/36 b)18/36 c)11/36 d)11/36 e)25/36 f)21/36 g)15/36 h)0 16.12)7; 6/36

Group 17: 17.1)a)4096 b)1/4096 c)2401/4096 d)1372/4096 e)294/4096 17.2)a)65,536 b)1/65,536 c)6561/65,536 17.3)a)1000 b)i)1/1000 ii)9/1000 iii)27/1000 iv)28/1000 17.4)a)14,641 b)10,000/14,641 c)40/14,641 d)4641/14,641 e)41/14,641 17.5)a)16.807 b)776,16,807 c)6480/16,807 d)2160/16,807 e)360/16,807 f)30/16,807 g)1/16,807 17.6)a)8000 b)24/8000 c)4896/8000 d)3104/8000 e)448/8000 17.7)a)50,625 b)90/50,625 c)18,720/50,625 d)21,600/50,625

CONDITIONAL PROBABILITY PROBLEMS: 1)a)1/8 b)4/8 c)6/8 d)3/8 e)7/8 f)1/64 g)1/4 h)2/5 i)0 j)3/5 2)a)2/3 b)1/4 c)1/7 3)a)1/5 b)2/6 4)a)40/250 b)160/250 c)90/250 d)50/120 e)70/110 f)40/130 5)a)4/12 b)0 c)12/48 d)4/40 e)2/20 6)a)1050/2300 b)2100/13800 7)a)9/40 b)4/30 c)24/75

Group 18: 18.3)a)$11.11;$10 b)$21.70;$20 c)$53.30;$50 d)$66;$60 e)$85;$80 f)$180;$170 g)$116.67;$110 h)$370;$350 18.4)a)1/18 b)1/9 c)1/2 d)1/2 e)7/12 f)1/18 g)5/18 h)0 i)1 j)1 k)1/3 l)5/9 m)4/7 n)0 18.5)a)3/38 b)35 to 3 c)$220 18.6)a)26/38 b)12 to 26 c)$100

Group 19: 19.1)0.6264 19.2)0.8103 19.3)0.0392 19.4)a)0.4615 b)0.2888 19.5)a)0.279 b)0.1507 19.6)0.3243 19.7)a)0.2305 b)i)0.6208 ii)0.0833 iii)0.2959 19.8)hockey

Group 20: 20.3)a)1287/2,598,960 b)65,780/2,598,960 c)22,308/2,598,960 d)24/2,598,960 e)4512/2,598,960 f)575,757/2,598,960 g)454,480/2,598,960 h)2,357,862/2,598,960 i)658,008/2,598,960 j)1,940,952/2,598,960

Group 21: 21.1)2,598,960 21.2)a)4 b)0.00000153 21.3)a)ace through 9 b)36 c)0.00001385 21.4)a)13 b)48 c)624 d)0.0002401 21.5)c)48 d)78 e)3744 f)0.001441 21.6)a)5148 b)5108 c)0.001965 21.7)a)40 b)256 c)10240 e)10,200 f)0.003925 21.8)a)54,912 b)0.02113 21.9)a)123,552 b)0.04754 21.10)a)1,098,240 b)0.4226 21.11)a)1,302,540 b)0.5012

Group 22: 22.1)a)48 b)9,10,J,Q,K,A c)12 d)8 e)24 22.2)a)1,712,304 22.3)a)128 b)0.00007475 22.4)a)128 b)0.00007475 22.5)a)336 b)0.0001962 22.6)a)16,800 b)0.009811 22.7)a)3168 b)2912 c)0.0017 d)480;0.0002803 e)1920;0.001121 f)512;0.000299 22.8)a)47,040 b)0.02747 22.9)a)16 b)4096 c)65,536 d)65,280 e)0.03812 22.10)a)215,040 b)0.1256 22.11)a)375,840 b)0.2195 22.12)a)858,240 b)0.5021 22.13)a)130,560 b)0.07625

Group 23: 23.1)a)82,160 b)prob: 0.014, 0.139, 0.847 c)0.153 23.3)a)1,581,580 b)prob:0.003, 0.043, 0.213, 0.741 c)0.259 23.4)a)300,500,200 b)prob: 0.00013, 0.00310, 0.02854, 0.12982, 0.83842 c)0.1616 23.5)a)146,107,962 b)1 in 146.107,962, 1 in 3,563,609, 1 in 584,432, 1 in 14,254, 1 in 11,927, 1 in 291, 1 in 745, 1 in 127, 1 in 69, 1 in 1.028 c)0.027 d)175,223,510 e)175,711,536 23.6)a)19,600 b)prob:0.006, 0.092, 0.902 c)0.098 23.7)a)12,103,014 b).0002481 c).05583 23.8)a)23,535,820 b).000001912 c).2709 d).04595

Group 24: 24.1)a)lose $0.50 b)lose $0.34 c)$0.50 d)$0.66 e)no, expect to lose 24.2)a)$0.40 b)yes, expect to win 24.3)a)12,271,512 b) 1 in b: 1 in 12,271,512; 1 in 48,696; 1 in 950; 1 in 53; 1 in 7; 1 in 1; expected value is: lose $0.66 c)lose $2409 d)$$1241 d)$3650 e)jar! 24.4) lose $0.83 24.5)a) win $0.17 b) $1.17 24.6)a)lose $0.32 b)lose $1.60 c)$0.18

Group 25: 25.1)a)1/38 b)$35 c)lose 5¢ 25.2)a)36/38 b)$17 c)lose 5¢ 25.3)lose 5¢ 25.4)a)34 to 4 b)$1 c)lose 5¢ 25.5)$0 25.6)a)5/38 b)$6 c)lose 8¢ 25.7)lose 5¢ 25.8)lose 5¢ 25.9)a)18/38 b)$1 c)lose 5¢ 25.10)true; to make $ 25.11)5 #'s; $0.08; $8.00 25.12) a) lose $0.17, b) no 25.13)$15 25.14)$4.11 25.15)win $0.65, so yes 25.16)a)lose $0.19, so no 25.17)a)lose $4.28 b)$0.72 25.18)$0 so it's fair

Group 26: 26.1)a)i)16/51 ii)4/51 b)2652 26.2)a)1326 b)64 c)0.0483 26.3)b)i)0.0478 ii)0.0476 iii)0.0475 26.4)a)32 b)AAA4, A222, AA32 c)AAAA3, AAA22 d)7 Aces 26.5)a)0 b)8 c)T=1.6; min bet d)T=4; max bet

TEST 2 REVIEW: 1)a)1/6 b)5/6 c)switch 2)a)66/1326 b)66 to 1260 3) 4:48 4) 16 to 36 5) 1/7 6)a)40/250 b)130/250 c)110/250 d)90/250 e)70/110 f)70/160 g)160/200 h)40/90 i)50/140 7)a)4/36 b)6/36 c)30/36 d)8/36 e)1/5 f)2/4 8)a)5/38 b)$6 c)lose $.08 d)$0 9)6/36 10) 243/1000 11)a)1/32 b)5/32 c)10/32 d)31/32 12)a)210/715 b)55/715 c)165/715 d)145/715 13) 7/26 14)a)36/91 b)55/91 c)28/91 d)63/91 15)a)35/5005 b)560/5005 c)4998/5005 16)225/455 17)a)0.2743 b)0.000022 c)0.02532 d)0.00175 18)a)win $1.66 b)yes, expect to win 19) -66¢ 20)a)6,5;2/6 b)3,2,1;3/6 c)4;1/6 21)a)straight b)3 of a Kind c)high card only 22)a)0.0255 b)0.000196 23)a)54,740 b)product column:0.16+ 0.21+ 0 = 0.37; expectation −$0.63 c)1/8.84 = 0.113 = 0.008 + 0.105 24)a)36/2,598,960 b)1,098,240/2,598,960 25)0.6346

FINAL EXAM REVIEW 1)a)sliced ham b)Bore c)5th 2)German 4)AA, BB, ABA, BAB, ABB, BAA 5)a)T b)F c)T d)T e)F f)F g)7 h)U i){ } j){f,h,i,k,l,p} k){e,f,g,h,j} l){ } m){e,g,j,k,l,p} n)128 o){ },{k},{l},{p},{k,l},{k,p},{l,p},{k,l,p} 6) $B^C \cap A$ 7){1,9} 8)a)i)nothing in A, nothing in B, but everything else ii)those who don't eat apples or cantaloupe b)i)only the two sections of C that are not in B ii)those who eat cantaloupe but not bananas 9)23 10)13 11)a)170 b)80 c)120 d)15 12)1,404,000 13)45 14)3024 15)924 16)384 17)5 to 8 18)37/256 19)a)16/52 b)16/52 c)28/52 d)4/40 e)26/39 20)1/11 21)a)0.2995 b)0.7785 c)0.04754 22)a)0.427 b)0.7798 c)0.2198 23)20/50 24)a)6/36 b)2/6 25)10/364 26)a)4/38 b)34 to 4 c)lose $0.05 d)$80 27)win $2.50 28)2596/3276 29)a)30/455 b)60/2730 30)0.8181 31)lose $0.85 32)a)142,506 b)0.001768 c)0.16 33)a)100,000 b)0.5905 c)0.0081 d)0.4095 34)switch

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