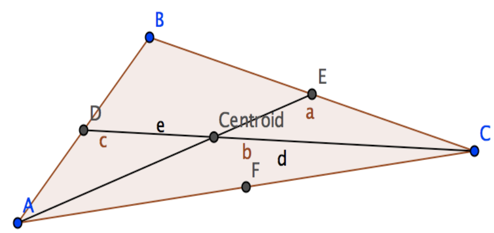
Centers of Triangles

Objective: To construct the four centers of a triangle: the centroid (medians), circumcenter (perpendicular bisectors of the sides), incenter (angle bisectors), and orthocenter (altitudes).

1. **Centroid**
   NY Standards: G.G. 43
   a. Start Geogebra or open a new window.
   b. Construct a triangle by going the **fourth** drop down menu and choosing **Polygon**.
   c. Click on three spots in the coordinate plane to create three points A, B, and C. Make sure to click back on point A.
   d. Go to the **second** drop down menu and choose **Midpoint.** Click on segment AB, segment BC and then on segment AC.
   e. Connect the midpoints D and E to their opposite vertices – this constructs the medians. Go to the third drop down menu and select **Segment between Two Points.** Click on point D then on point C and click on point E then point A.



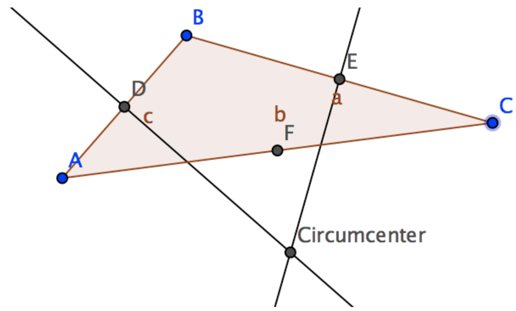
   f. Locate the point of intersection. Go to the second drop down menu and choose **Intersect Two Objects.** Label this point **Centroid.**
   g. On the far left are two lists (if Algebra View is selected in the **View** menu): Free Objects and Dependent Objects. Point G (the centroid) is in the Dependent list. Double click on G and a dialog box will come up. Choose the **Object Properties** button. Under the **Basic** menu, change the Name from G to Centroid and click Close.
   h. Verify that the third median goes through this point. Also go to the first down drop menu and choose **Move**. Move a point (A, B, or C) around to see what happens to the centroid.
   i. The Centroid is called the center of mass of the triangle. If you were to cut out this triangle, you could balance it on the head of a pin placed at the Centroid. Can the Centroid be outside the triangle? Why or why not (explain in a text box; go to the tenth drop down menu and choose **Insert text**)?
   j. Explain as best you can why you think the intersection of the lines you've constructed is the centers of mass (Centroid) in the text box. You might do a bit of online searching to see what discussions there are about this topic.

2. **Circumcenter**
   a. Hide the three medians and Centroid point. DO NOT DELELTE, we will need these later! Go to the last drop down menu and choose **Show/Hide**

Objects. Click on the medians and the Centroid (each should become darker grey) and then click on the first drop down menu. The triangle ABC and three midpoints should be showing.

b. Through two midpoints, construct perpendicular bisectors to two sides. Go to the fourth drop down menu and choose **Perpendicular Bisector.** Click on segment AB and segment BC.

c. Locate the point of intersection. Label this point **Circumcenter**. Verify that the third perpendicular bisector goes through this point.



d. To see why this is called the Circumcenter. Go to the sixth drop down menu and choose **Circle with Center through Point**. Select the Circumcenter and point A. You will see that the circumcenter is the center of the circle that circumscribes the triangle.

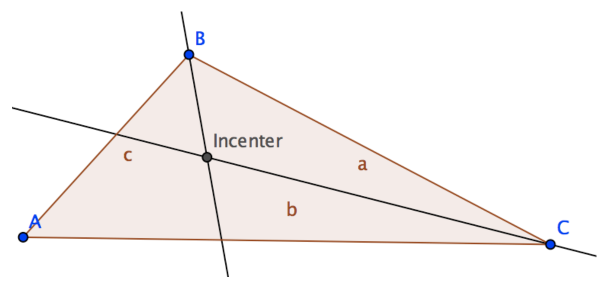
e. Can the circumcenter be outside the triangle? Why or why not (explain in the text box)?

f. Explain in the text box as best you can why you think the intersection of the lines you've constructed form the center of a circle that circumscribe the triangle. You might do a bit of online searching to see what discussions there are about this topic.

3. **Incenter**

a. Hide all of the lines and points. DO NOT DELELTE, we will need these later! Only triangle ABC should be left.

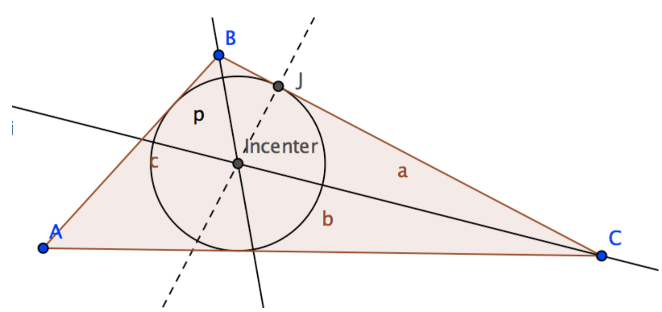
b. Go to the fourth drop down menu and choose **Angle Bisector.** Click on three points on the triangle (i.e. A, B, C). Repeat this on a second angle.



c. Locate the point of intersection. Label this point **Incenter**. Verify that the third angle bisector goes through this point.

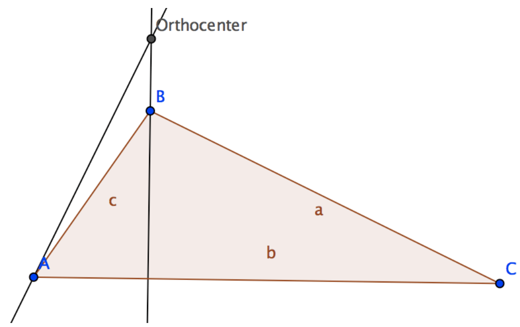
d. To see why this is called the Incenter, go to the fourth drop down menu and choose **Perpendicular Line**. Click on Incenter and segment BC. Locate the point where the perpendicular line from the Incenter intersects segment BC (the perpendicular line is the dotted line in the figure). The point of intersection is labeled J.

e. To see why this point is called Incenter, go to the sixth drop down menu and choose **Circle with Center through Point.** Select the Incenter and the intersection point J**.** You will see a circle that is completely contained inside the triangle and each side of the triangle is tangent to the circle.



f. Will the Incenter be outside the triangle? Why or why not (explain in the text box)?
g. Explain (in the text box) as best you can why you think the intersection of the lines you've constructed form the center of a circle that is inscribed in the triangle. You might do a bit of online searching to see what discussions there are about this topic.

4. **Orthocenter**
   a. Hide all of the lines and points. DO NOT DELELTE, we will need these later! Only triangle ABC should be visible.
   b. To create the altitudes of the triangle, go to the fourth menu and choose **Perpendicular Line.** Click on segment AC and the point opposite it (point B). Repeat for vertex A and segment BC.

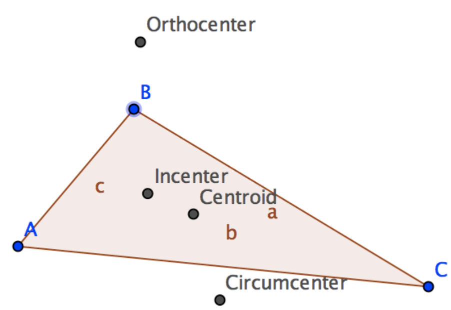


   c. Locate the point of intersection. Label this point the **Orthocenter**. Verify that the third altitude goes through this point.
   d. Can the orthocenter be outside the triangle? Why or why not (explain in the text box)?

5. Euler Segment

This exploration uses the triangle centers constructed above.

a. Hide all the lines and segments. Continue until there are no lines left sticking out from the triangle. Now select the three medians and the three midpoints of the sides. Hide them, as well.



b. You should now have only the four centers (Centroid, Circumcenter, Incenter and Orthocenter) and the triangle remaining.
c. To make the centers easier to see re-label the Centroid as CE, the Circumcenter as CC, the Incenter as IC and the Orthocenter as OC.
d. Drag a vertex of your triangle around. Which centers appear to be collinear (answer in the text box)?
e. Are these centers collinear for any triangle (make various measurements to determine which type of triangle you have)?
f. Are there any triangles where all four centers appear to be collinear?
g. Are there any triangles where the four centers are the same point?
h. Connect the Orthocenter, OC, and the Circumcenter, CC, of your triangle with a segment. This is called the Euler Segment. Now drag around one of the vertices of the triangle. What do you notice (answer in the text box)?
i. If you want to try to animate the Euler Segment, then vertex B needs to be a slider. Go to http://www.mathcasts.org/mtwiki/GgbHelp/Animate for the directions. It is worth trying!