Concrete Conics and Pencils in Projective Geometry

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Projective Geometry

- "Universal" geometry for Euclidean, Hyperbolic, and Elliptic Geometries

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- For Students, many concepts seem non-intuitive and **weird**!

Weird Ideas for Students to Conceptualize

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- Conics - why are they "natural"?

Transformational and Axiomatic Foundation

Projective Geometry has a ''Natural" Axiomatic Structure

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- Affine Geometry:
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- Projective Geometry:
  Incidence, Transformations Preserving Incidence

Projective Geometry - Incidence

Given: Points, Lines, What are natural ''Incidence" notions?

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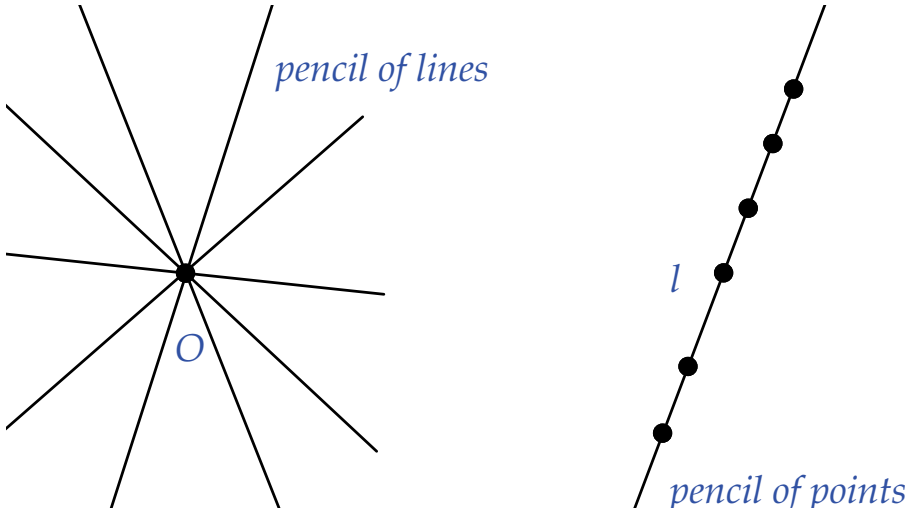
- Points on Lines
- No Parallels $\rightarrow$ lines have no ''special" status

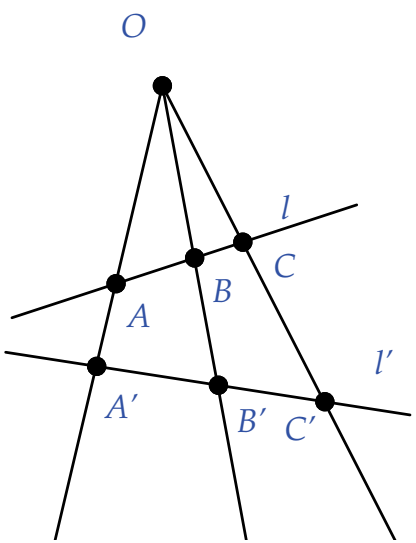
Projective Geometry - Pencils



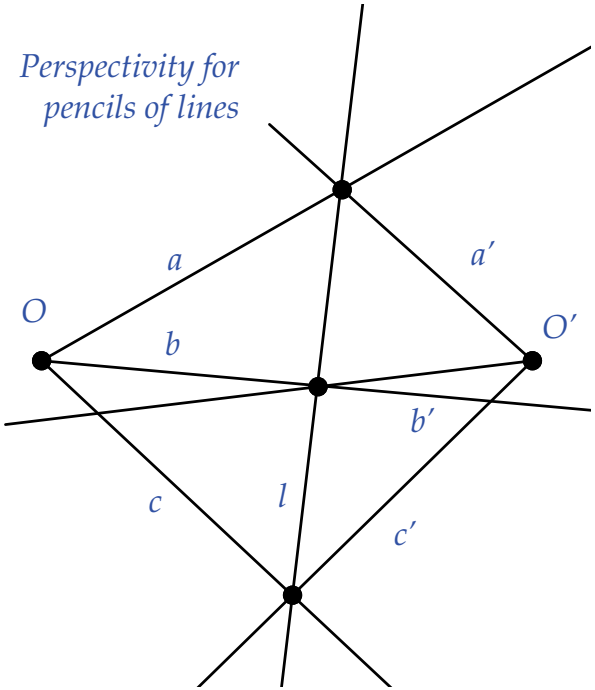
Pencil = set of **co-incident** elements (lines, points)

Projective Transformations - Preserve Incidence

Perspectivity for pencils of points



Perspectivity for pencils of lines



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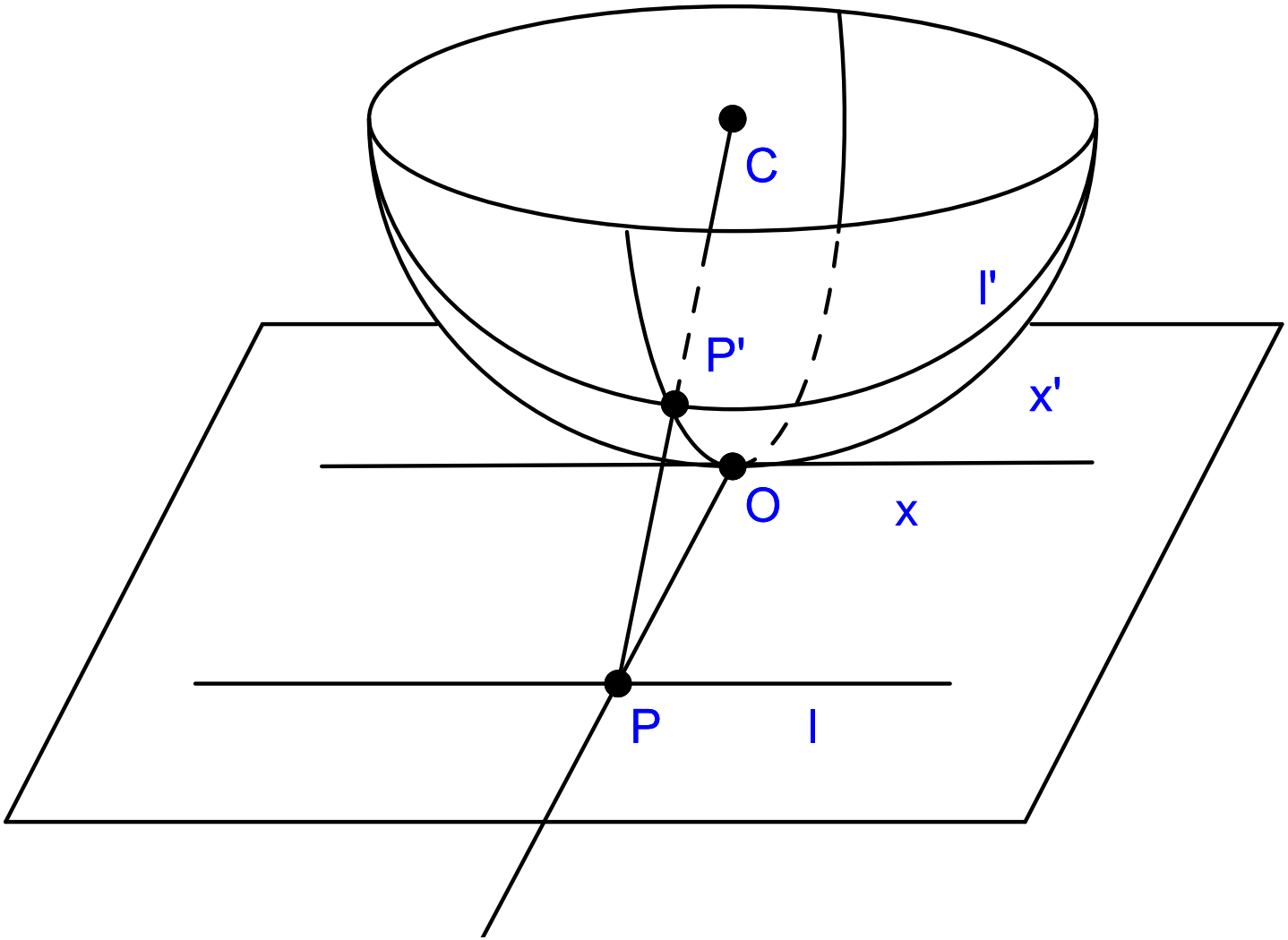
Let's explore!

Central Projection Model of Euclidean Geometry



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- Software can provide laboratory for exploring fundamental ideas such as conics, pencils, and projectivities
- **Central Idea**: Intuitive Development $\rightarrow$ Deeper Conceptual Understanding